Constraint Programming and Local Search

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Outline

1. Introduction
2. A didactic optimization problem (dTP)
   - Motivations for cooperation
3. A zoo of CP / LS hybrids
   - Sequential combination
   - *Master / sub-problem decomposition*
   - Improved neighborhood exploration
   - CP Neighborhood search
   - Large neighborhood search
   - Local moves during construction
   - *Local moves over a heuristic*

What this tutorial addresses

- Solving large hard combinatorial optimization problems
- Systematic description of ways of combining LS and CP techniques
- Goal: provide a check-list of recipes that can be tried when tackling a new optimization application
- Illustrated on a didactic problem

When should you enquire about CP / LS hybrids ?

- When you have:
  - A large complex optimization problem
  - No solution neither with CP nor with LS
  - The problem specification may change over time
- Best in case of strong execution requirements
  - Limited planning resource
  - On-line optimization

When can’t it help ?

- When modeling is the issue
- When optimization is the single difficulty
- When thousands of man.year have been spent studying your very problem

  => useless for solving a 1M node TSP

Comparing CP and LS

- Constraint Programming
  - Solves complex problems
  - Models capturing many side constraints
  - Solves by global search and propagation
- Local search
  - Solves problems with simple models
  - Efficiency: quick first solution, rapid early convergence

Opportunities for collaboration

- Expected combination of :
  - Generality (solve complex problems)
    - Nice modeling
    - Generic methods from the model
    - Easy to add/modify constraints
  - Efficiency (solve them fast)
    - Initial solution
    - Quick convergence
  - Address both feasibility and optimization issues
    - keep constraints hard
- Difficulty to combine:
  - monotonic reasoning (CP)
  - non-monotonic modifications (LS)

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A didactic transportation problem

Collect goods from clients

- Set of trucks located in a depot
- Each truck can carry two bins
- Each bin may contain only goods from the same type
- Clients have time window constraints
- Bins have capacity constraints

A simple model for dTP

$i,j \in \{1, \ldots, n\}$: clients (their locations)

$k \in \{1, \ldots, M\}$: trucks

$h \in \{1, \ldots, 2M\}$: bins

$l \in \{1, \ldots, P\}$: types of goods

Model

Minimize $totCost = \Sigma_{k=1}^{M} cost_k$

On $\forall k,\ cost_k \geq 0$

$truck_k$: UnaryResource$(tt,c,cost_k)$

$\forall h,\ collects_h \in [1 .. P]$

$\forall i,\ start_i \in [a_i .. b_i]$

$service_i$: Activity$(start_i,d_i,i)$

$visitedBy_i \in [1 .. M]$

$collectedIn_i \in [1 .. 2M]$

Model (2)

Subject to

$$\forall i,\ service_i \text{ requires } truck[visitedBy_i]$$

$$\forall h,\ \Sigma_{i \mid collectedIn\ i = h}\ q_i \ \leq C$$

$$\forall i,\ collects[collectedIn_i] = type_i$$

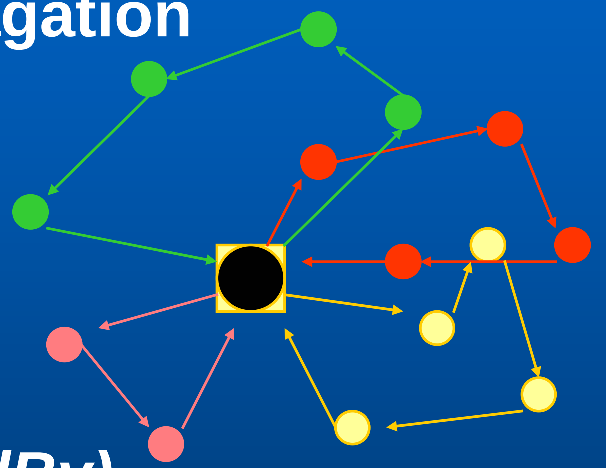
$$\forall i,\ visitedBy_i = \lceil collectedIn_i / 2 \rceil$$

A CP approach

- Strengthen the model
  - Add redundant constraints
  - Add global constraints
  - Add constraints evaluating the cost of the solutions
  - Symmetry breaking (dominance) constraints

- Find a search heuristic
  - Variable / value orderings
  - Explore part of the search tree through Branch and Bound

A CP approach



Redundant models for stronger propagation

Example: redundant routing model

$\forall k, \qquad first_k \in [1 .. N]$

$\forall i, \qquad next_i \in [1 .. N+M]$

$\qquad succ_i \in [\{\} .. \{1, \ldots, N\}]$

multiPath(first,next,succ,visitedBy)

costPaths(first,next,succ,c,totCost)

$\forall i,j, \quad j \in succ_i \Leftrightarrow$

$\qquad (visitedBy_i = visitedBy_j \wedge start_i < start_j)$

Solving through CP

- Instantiate $visitedBy_i$
- Rank all activities on the routes
  (instantiate $next_i$ / $succ_i$)
- Instantiate $start_i$ to their earliest possible value

Difficulties with CP

- Poor global reasoning
- Poor cost anticipation
- Goes backtracking « forever »
- As propagation is strengthened, the model is slowed down

A local search approach

- Two possibilities:
  - Work in the space of feasible solutions
  - Accept infeasible solutions by turning constraints into penalties
- Possible combinations, work with feasible but add, if needed, extra resources (trucks and bins)

Local search for dTP

- Generate an initial solution
  - Select clients *i* in random order
  - Assign it to a truck that has a bin of $type_i$, or to a truck that can be added an extra bin of $type_i$
- Move from a solution to one of its neighbors, in order to improve the objective

Neighborhoods for dTP

- Node transfer:
  - Change values of $visitedBy_i$ and $collectedIn_i$ for some i

- Bin swap:
  - Select bins h_1, h_2 on trucks $k_1 = \lceil h_1/2 \rceil$, $k_2 = \lceil h_2/2 \rceil$
  - For all clients i,

    $collectedIn_i = h_1 \Rightarrow collectedIn_i = h_2, visitedBy_i = k_2$

    $collectedIn_i = h_2 \Rightarrow collectedIn_i = h_1, visitedBy_i = k_1$
  - Swap $collects_{h1}$ and $collects_{h2}$

Neighborhoods

- *k*-opt:
  - select $i1, i2, i3$ such that $visitedBy_{i1} = visitedBy_{i2} = visitedBy_{i3}$
  - Exchange edges:
    - Replace $next_{i1}=j1$, $next_{i2}=j2$, $next_{i3}=j3$
    - By $next_{i1}=j2$, $next_{i2}=j3$, $next_{i3}=j1$

Driving the local search process

- Main iteration:

  Until a global stopping criterion is met:
  - generate a new initial solution
  - perform a local walk
- Each walk:

  Until a local criterion is met:
  - Iterate the neighborhood, until a neighbor satisfying all constraints as well as the acceptance criterion is found
  - Perform the move

Difficulties with LS

As the problem gets more constrained…

- Generating a good feasible first solution becomes harder
- Exploring neighborhoods
  - takes longer: constraints checks
  - is less interesting: fewer valid nodes
  - more local optima appear

Conclusion

Neither of the “pure” approaches works

- Need for hybridization with other techniques
  - Try a cooperation between CP and LS
  - Expect to retain :
    - *Good sides of CP*: handling side constraints, building valid solutions, systematic search
    - *Good sides of LS*: quick easy improvements, quick convergence.

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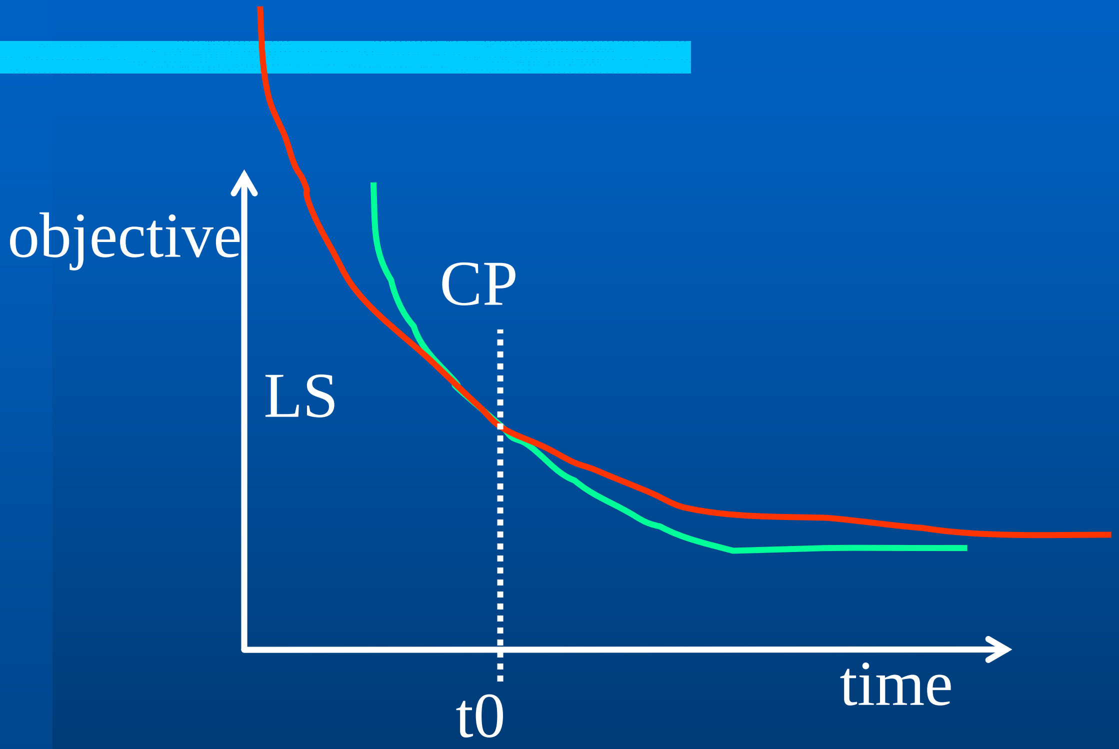
Sequential combination: LS-CP



- Use local search for the beginning of the optimization descent – switch to CP at time t0

Discussion

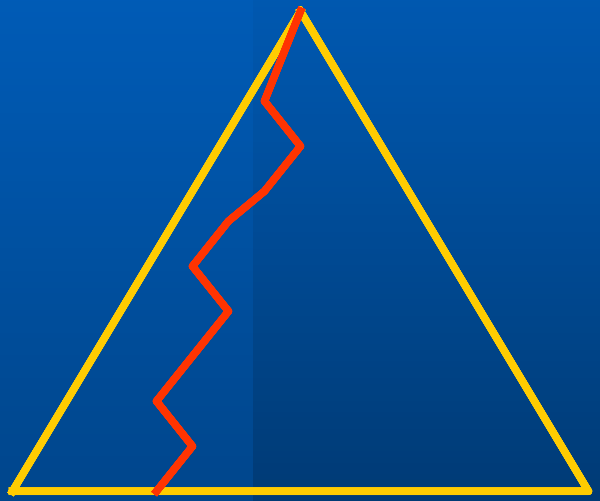
- A good idea
  - When the feasibility problem is easy
  - For time-constrained optimization
- But, the switch from LS to CP is not immediate
  - CP starts with a good upper bound, but without no-goods
- On the didactic Transportation Problem (dTP)
  - Lack of good lower bounds

    => Systematic CP search gets stuck near the optimal region

Sequential combination: CP-LS

- Build a first feasible solution with CP
  - Greedy heuristic
- Try to improve it through LS
  - Constraints can be softened to support dense neighborhoods

Greedy insertion algorithm



- At each choice point a function *h* is evaluated for all possible choices
- The choice that minimizes *h* is considered as preferred decision
- The preferred decision is taken

Greedy insertion for dTP

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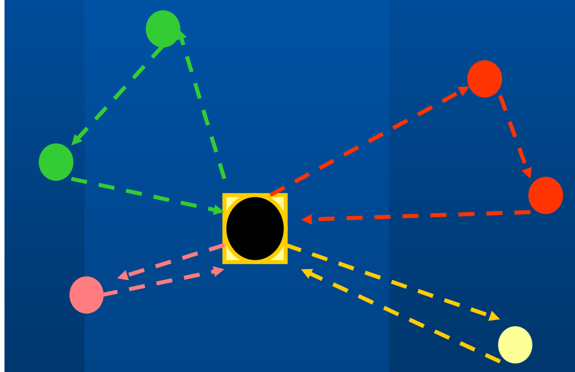
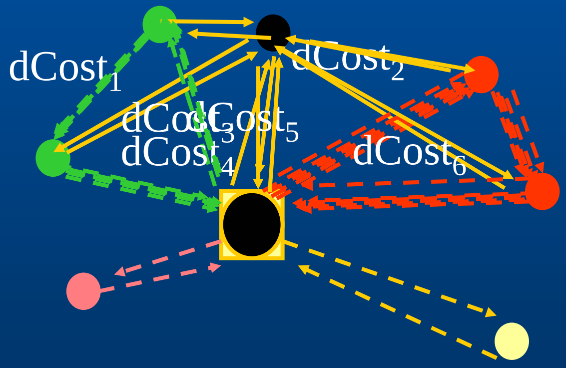
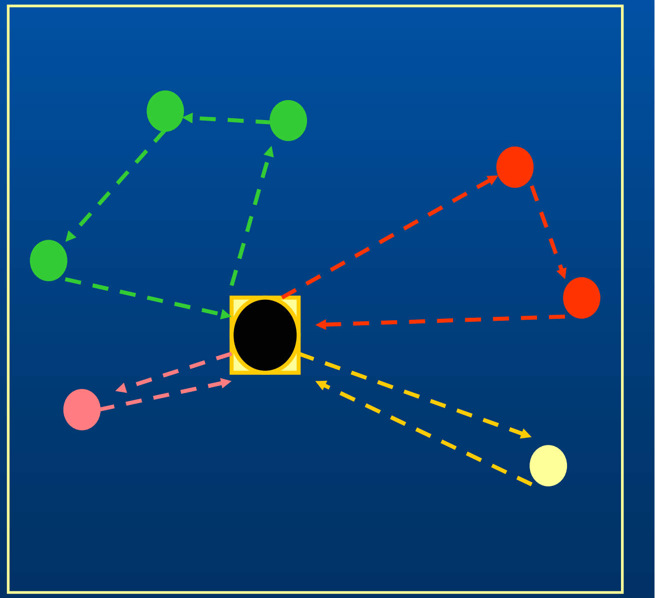


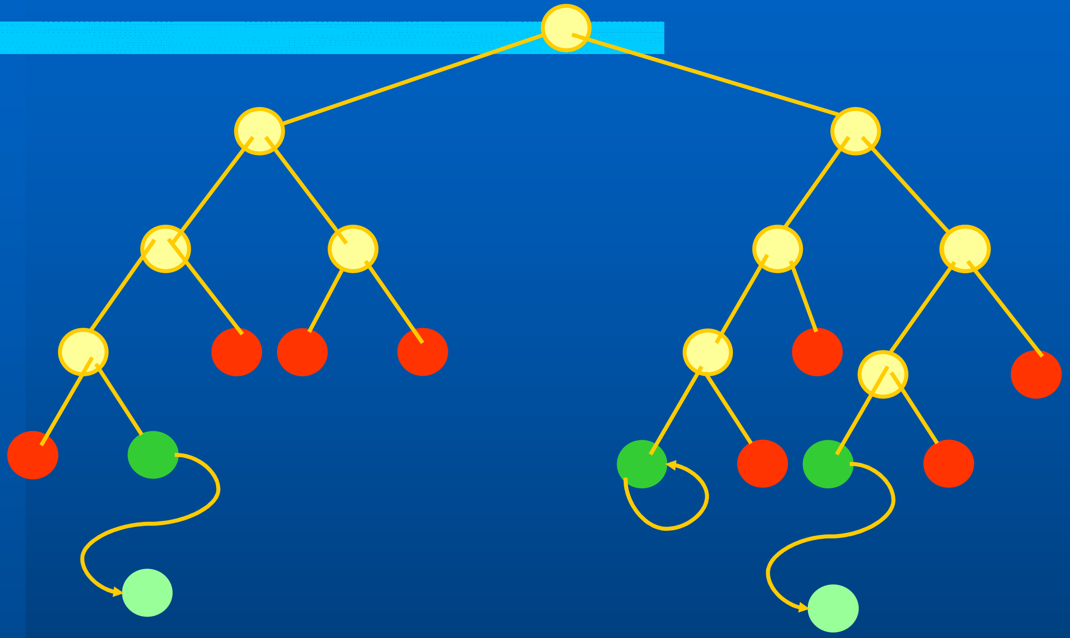




Discussion

- CP then LS: can be interesting for dTP
  - In particular in case of tight side constraints
- « One-shot » use of CP:
  - as long as no valid solution has been found, we look for one
  - Enables to start LS with a valid solution

A systematic combination



- Solve the problem through CP (global search tree)
- Try to improve each solution found through local search
- Improve the optimization cuts

Discussion

- Local moves should change the assignment of « early » variables
  - Avoid visiting the same region as with backtracking
- Especially interesting in case of incomplete search
  - CP provides a set of diversified seeds for local search

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Master / sub-problem decomposition

- Idea: identify two sub-problems and solve them by different techniques
  - Master problem
  - Induced sub-problem
- Decomposition: the sub-problem can only be stated once the master problem is solved.

Purpose of decomposition

- Decompose into easier problems
  - Smaller size
  - Simpler models
  - Well known structure
- Traditional approach with exact methods (Dantzig, Lagrangean, Benders)

A decomposition on dTP

- Master Problem:
  - Assignment of clients to trucks (*visitedBy)*
- Induced sub-problem:
  - Traveling salesman with time windows

- Algorithms:
  - Assess a cost for each client (e.g. distance to neighbor), solve assignment with some method
  - Solve small TSPs with CP
  - Analyze TSPs, re-assess client cost and try improving local moves on the master problem.

Discussion

- Decomposition makes the problem easier to solve
- Estimating the cost in the master problem may be difficult
- Try local changes on the evaluated cost of the master problem
  - improve subsequent optimization (feedback from the sub-problem)

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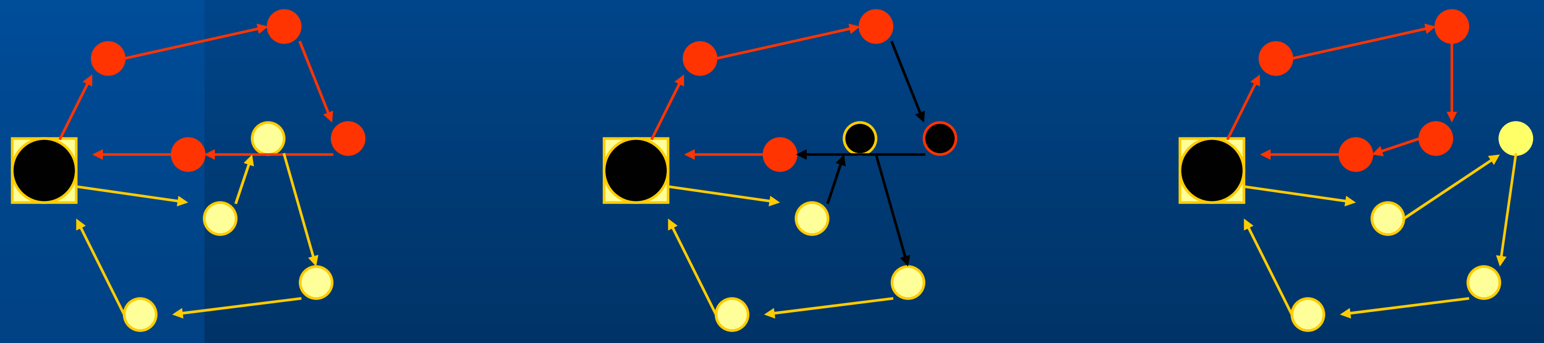
Constrained local search

- Small neighborhoods
  - A neighbor solution S_1 can be reached from a given solution S* by performing "simple" modifications of S*.
  - Examples:
    - Choose two visits i_1 and i_2, remove i_1 from its current position and reinsert it after i_2
    - Choose two visits i_1 and i_2 and exchange their positions

Constrained local search

- Node Exchange
  - Choose two visits i_1 and i_2 assigned to different trucks and exchange their positions
  - Accept the first exchange improving the cost

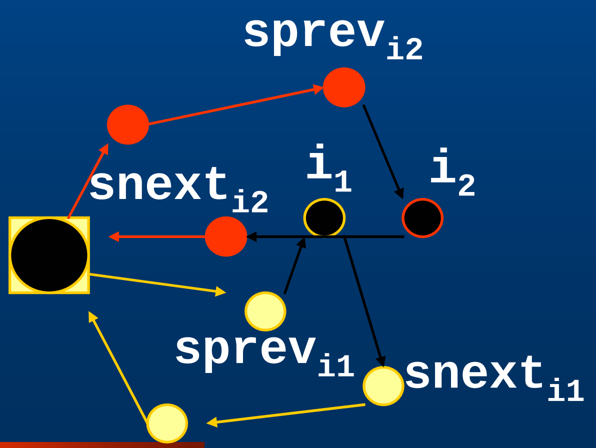


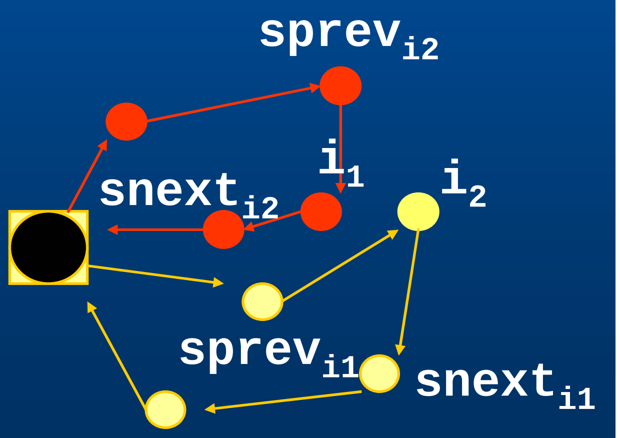
Node Exchange: version 1

```
Procedure exchange(P,S)
   forall nodes $i_1$
      forall nodes $i_2$ | ($svisitedBy_{i1} \neq svisitedBy_{i2}$ )
         exchangeInstantiate(P,S,$i_1$,$i_2$)
         // check feasibility and check cost function
         if (propagate(P) && improving(P,S))
            storeSolution(P,S)
            resetProblem(P)
            exit iterations
         // reinitialize the domain variables
         resetProblem(P)
```

Node Exchange: version 1

```
Procedure exchangeInstantiate(P,S,$i_1$,$i_2$)
    // exchange $i_1$ and $i_2$
    next[$sprev_{i1}$] = $i_2$; next[$i_2$] = $snext_{i1}$;
    next[$sprev_{i2}$] = $i_1$; next[$i_1$] = $snext_{i2}$;
    // restore the rest
    forall k $\notin$ {$i_1$,$i_2$,$sprev_{i1}$,$sprev_{i2}$}
        next[k] = $snext_k$;
```





Node Exchange: version 1

- Pros:
  - Independent from side-constraints

- Cons:
  - CP imposes monotonic changes
    - ➔ while moving from one neighbor to the next one all problem variables are un-instantiated and re-instantiated
  - Constraints are checked in “generate and test”
    - ➔ inefficient

Node Exchange: version 2

Add inlined constraint checks

```
Procedure exchange(P,S)
   forall nodes $i_1$
     forall truks k | (k $\neq$ $svisitedBy_{i1}$ )
        if (not binCompatible(P,S,$svisitedBy_{i1}$,k)) continue
        forall nodes $i_2$ | ($svisitedBy_{i2}$ = k)
           if (not timeWindowCompatible(P,S,$i_1$,$i_2$)) continue
           if (not improving(P,S,$i_1$,$i_2$)) continue
           exchangeInstantiate(P,S,$i_1$,$i_2$)
           // check feasibility && check cost function
           if (propagate(P) && improving(P,S))
              storeSolution(P,S)
              resetProblem(P)
              exit iterations
           resetProblem(P); // reinitialize the domains
```

Node Exchange: version 2

- Pros:
  - “Almost” independent from side-constraints
  - Some constraints are tested before performing the move

    → much more efficient
- Cons:
  - CP imposes monotonic changes
  - Some constraints are still checked in “generate and test”

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CP Based Operators

- Operators define neighborhoods
- Finding the best solution in a neighborhood is an optimization problem
- Which can be solved with constraint programming
- Neighborhood search problem can be expressed:
  - With a specific model and interface constraints
  - With the original model and additional constraints

Specific Model

- A special model is developed to represent the neighborhood
- Interface constraints link the new model to the original model
- All the constraints stated in the original model are enforced in the specific model via the interface constraints
- During search, constraint propagation allows to prune (via the interface) regions of the neighborhood
- No restrictions on the neighborhood which can be defined

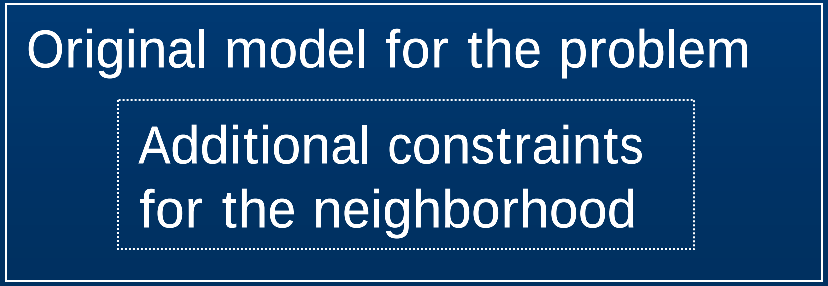

Original model for the problem

Interface constraints

Specific model to define the neighborhood

Original Model

- The neighborhood is defined simply by adding additional constraints to the original model
- No need to define a new model and interface constraints
- All the constraints in the original model are naturally enforced
- During search, constraint propagation allows to prune directly large regions of the neighborhood
- Not all neighborhoods can defined inside the original model (i.e. GENeralized Insertion)



Original model for the problem

Additional constraints for the neighborhood

Node Exchange

CP based neighborhoods

- The neighborhood of a solution S for a problem P is defined by a constraint problem

$$NP(P,S) :: [\{I_1,\ldots,I_n\},\{C_1,\ldots,C_m\}]$$

- Each solution of NP represents a neighbor of S for P

Node Exchange: nhood model

- **Variables: I::[0..n-1], J::[0..n-1], DCost::[-∞..0]**

  I, J are domain-variables representing the nodes *i,j* that we want to exchange.

- **Constraints:**

// neighborhood cst

I > J

svisitedBy[I] ≠ svisitedBy[J]

next[I] = snext[J]

next[J] = snext[I]

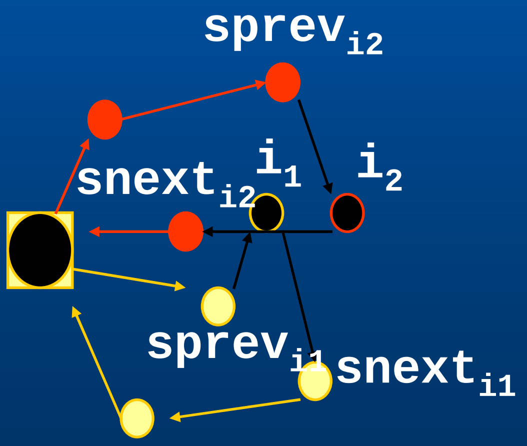
next[sprev[I]] = J

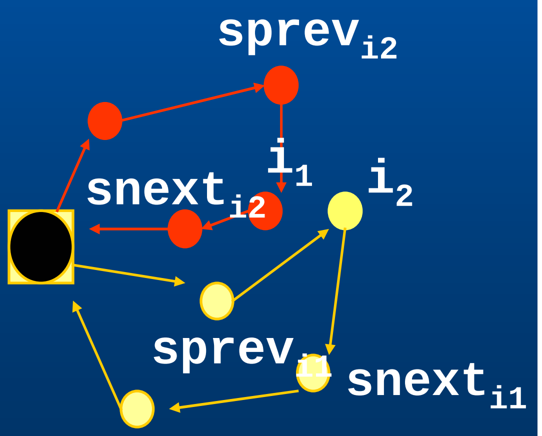
next[sprev[J]] = I

// interface cst

forall k, (k ≠ I ∧ k ≠ J)

⇒ next[k] = snext[k], visitedBy[k] = svisitedBy[k]





Node Exchange: nhood model

- DCost represents the gain w.r.t S:

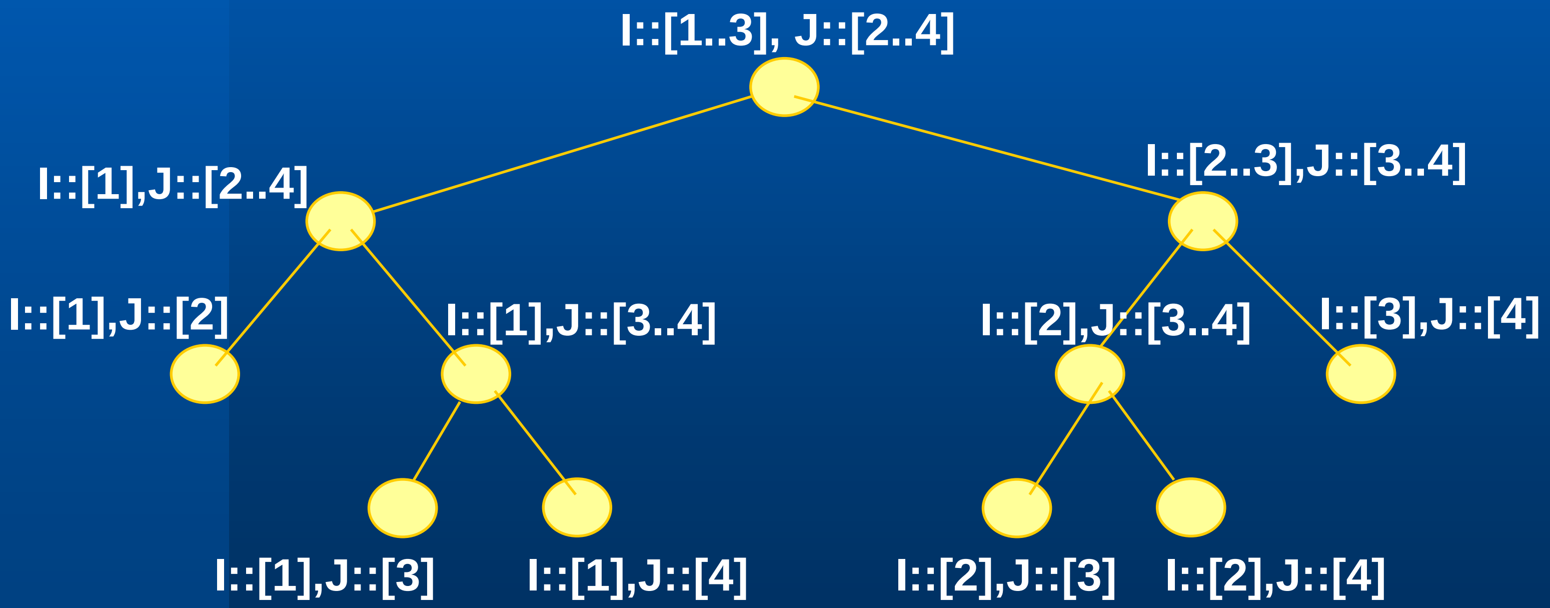
```
DCost = cost[sprev[J], I] + cost[I, snext[J]] +
            cost[sprev[I],J] + cost[J,snext[I]] -
            cost[sprev[I], I] - cost[I,snext[I]] -
                      cost[sprev[J],J] - cost[J,snext[J]]
// improving cst
DCost < 0
```

- Search (explore via tree search):

  instantiate(I) && instantiate(J)

Node Exchange: nhood model

- Search: instantiate(I) && instantiate(J)
- Each leaf defines a feasible exchange



I::[1..3], J::[2..4]

I::[1],J::[2..4]

I::[2..3],J::[3..4]

I::[1],J::[2]

I::[1],J::[3..4]

I::[2],J::[3..4]

I::[3],J::[4]

I::[1],J::[3]

I::[1],J::[4]

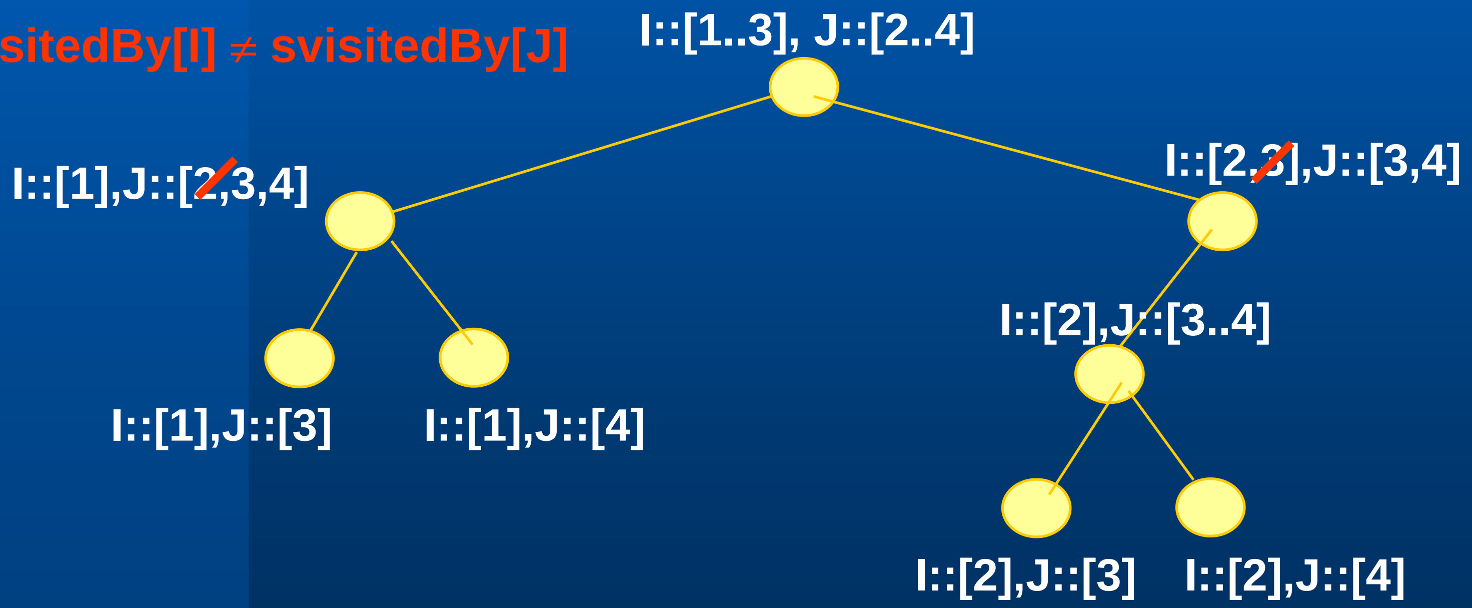
I::[2],J::[3]

I::[2],J::[4]

Node Exchange: nhood model

- Suppose that in S:
  - clients 1,2 are visited by truck 1,
  - clients 3,4 are visited by truck 2

svisitedBy[I] ≠ svisitedBy[J]



I::[1..3], J::[2..4]

I::[1],J::[~~2~~,3,4]

I::[2,~~3~~],J::[3,4]

I::[1],J::[3]

I::[1],J::[4]

I::[2],J::[3..4]

I::[2],J::[3]

I::[2],J::[4]

Node Exchange: nhood model

- Pros:
  - Independent from side-constraints
  - Constraint Propagation removes infeasible neighbors a priori.
    - ➔ efficient when many side constraints
    - ➔ efficient when large neighborhoods
  - May freely mix tree search and local search
- Cons:
  - Overhead due to tree search

Node Exchange: nhood model

- Overhead due to tree search
  - Often most problem variables are instantiated by the interface constraints only when ALL neighborhood variables are instantiated (at every leaf of the nhood tree search)
  - In this case the nhood tree search keeps “doing” and “undoing” the instantiations of ALL the problem variables

Local Search via solution deltas

- **Goal: avoid instantiating and un-instantiating ALL problem variables while moving from one neighbor to the other**
  - **A neighbor is identified by the modification over the original solution S. This modification is defined *solution delta.***
  - **A neighborhood is an array of *deltas.***
  - **The exploration of the neighborhood takes place on a tree search.**

LS via solution deltas : Node Exchange

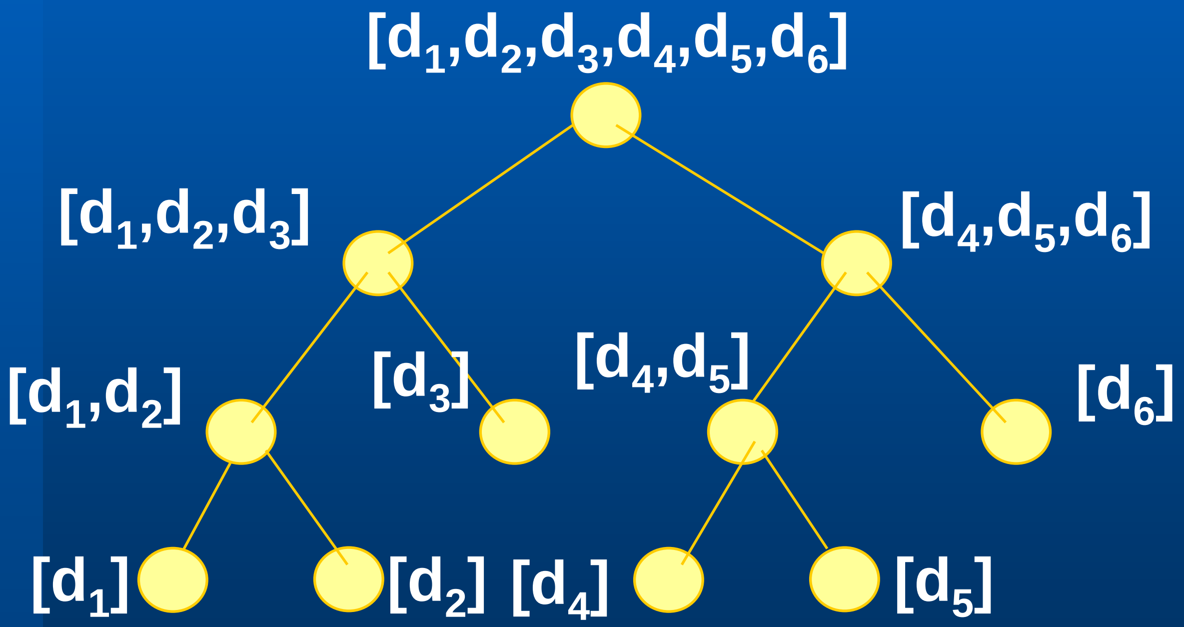
```
Procedure exchange(P,S)
   SolutionArray neighborArray
   forall nodes $i_1$
     forall nodes $i_2$ | ($svisitedBy_{i1} \neq svisitedBy_{i2}$ )
       Solution delta = {(next[sprev[$i_2$]] = $i_1$),
                         (next[$i_1$] = snext[$i_2$]),
                         (next[sprev[$i_1$]] = $i_2$),
                         (next[$i_2$] = snext[$i_1$])}
       neighborArray.add(delta)
   exploreNeighborhood(P,S,neighborArray)
```

LS via solution deltas: explore the neighborhood

- Map the array of deltas in a tree search
  - recursively split the array of deltas in two parts
  - a split correspond to a branching node in the tree search
  - each feasible neighbor is a leaf of the tree
  - at each node restore the fraction of S that is shared by all neighbors in that node

Exploring the neighborhood through solution deltas

- Example: *deltas* = $[d_1,d_2,d_3,d_4,d_5,d_6]$



Local Search via solution deltas

- Pros:
  - Independent from side-constraints
  - Constraint Propagation removes infeasible neighbors a priori.
    - ➔ efficient when many side constraints
    - ➔ efficient when large neighborhood
  - May freely mix tree search and local search
  - Reduced overhead of the tree search
- Cons:
  - Requires an explicit generation of the neighborhood
  - Requires to fully specify each move

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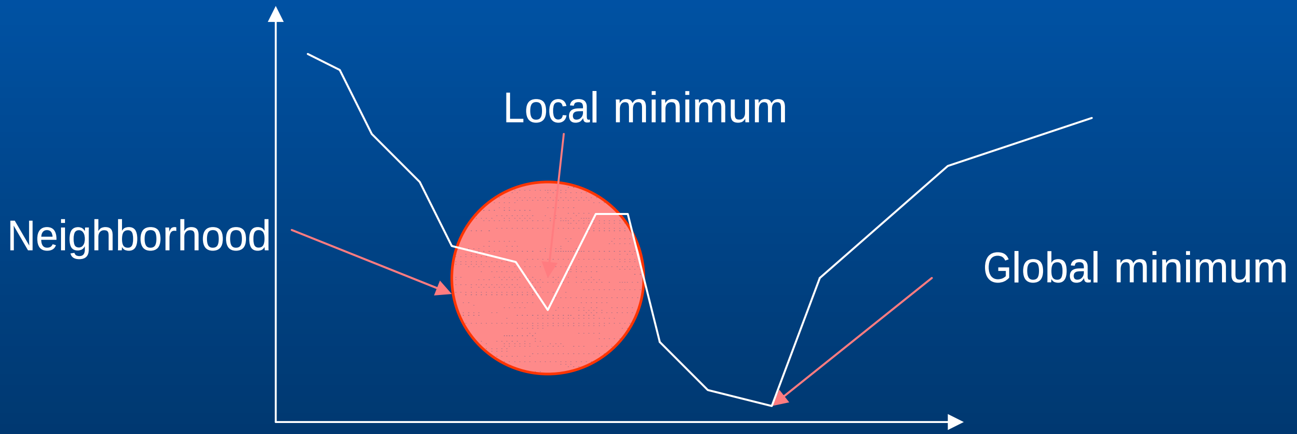
LS and Local Minima

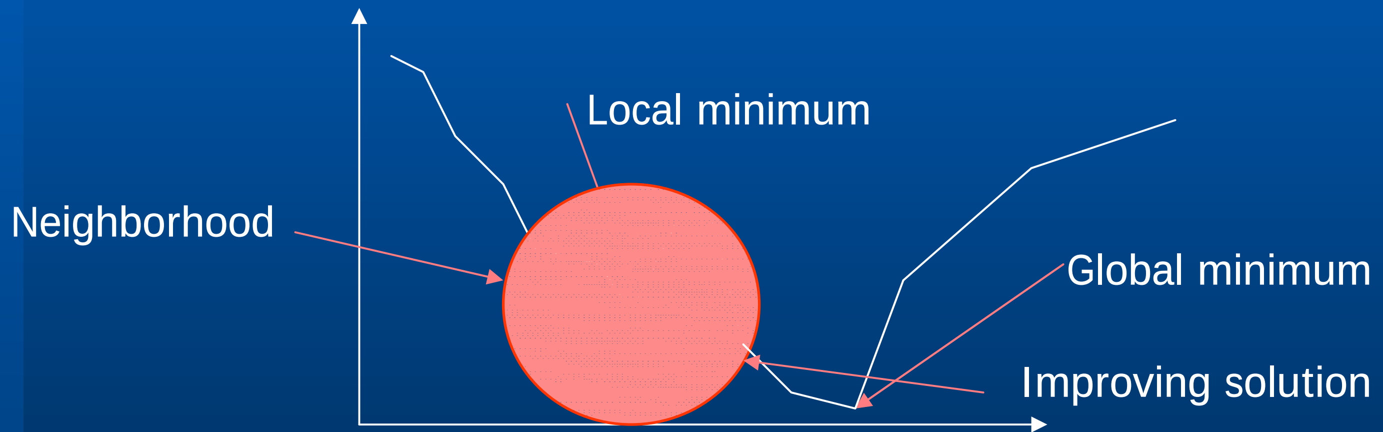
- A local minimum is reached when no solutions in the neighborhood is better than the current solution



- Usual solution is to use metaheuristics to allow a temporary degradations of the objective

Large Neighborhoods: Gains

- A larger neighborhood means:
  - More solutions are considered
  - Better chance of avoiding local minima



  - Can still use metaheuristics

Large Neighborhoods: loss

- **A larger neighborhood also means:**
  - **More solutions need to be evaluated**
  - **The complexity of evaluating all solutions makes having neighborhoods too large unattractive**

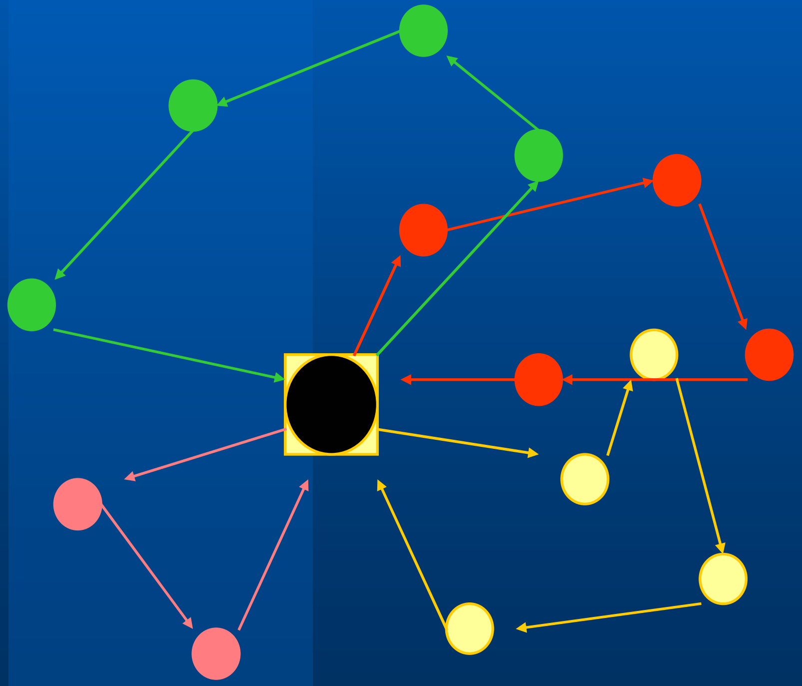
  - Unless we don’t evaluate all the solutions !
    - This is were Constraint Programming is useful

Large neighborhood search

- Idea: partition the variables of the current solution into two subsets
  - A fragment: assignments are kept as they are
  - A shuffle set: assignments may be changed

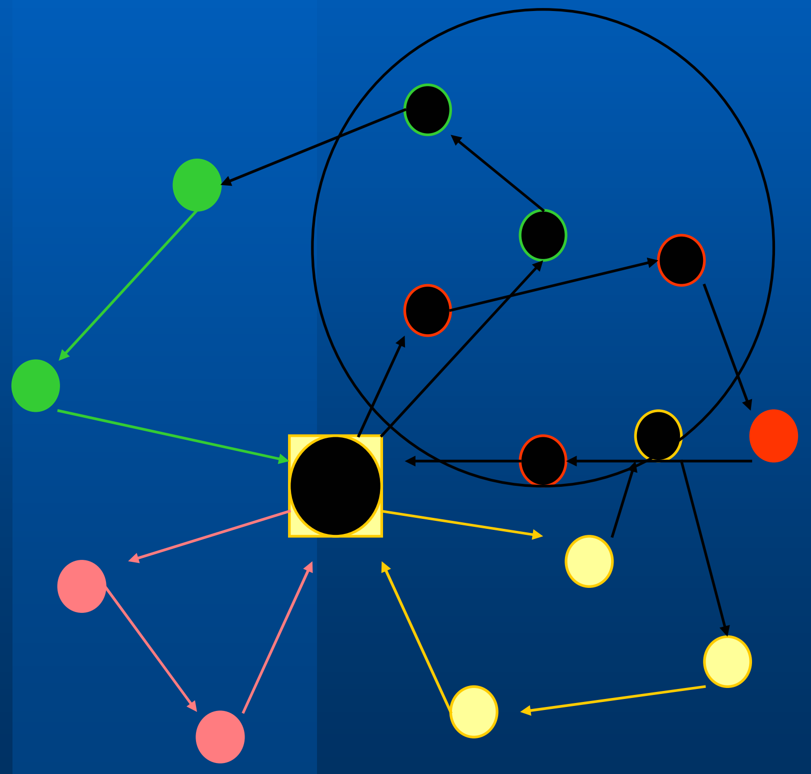
Large neighborhood search on dTP

- From a solution



Large neighborhood search on dTP

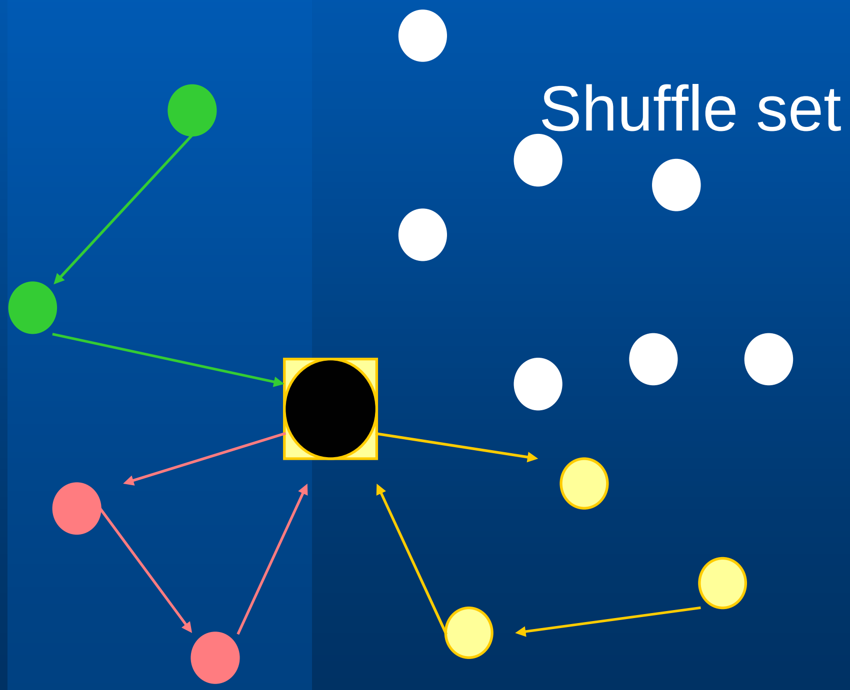
- Select a shuffle set



Select a subset of clients $i_1, i_2, \ldots, i_k \subset C$

Large neighborhood search on dTP

- Example on dTP

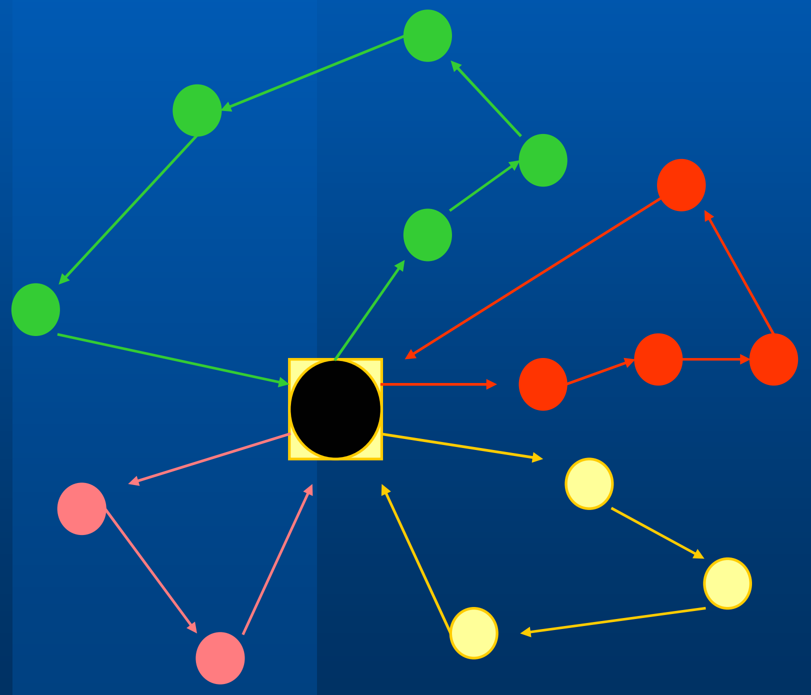


For all clients *i* from the shuffle set :

- Unassign variable:
  - $visitedBy_i$, $collectedIn_i$
  - $start_i$
- Undo all ordering decisions between *i* and other clients *j*
- Unassign all cost variables
- Post a cost improvement cut

$$cost \leq getValue(cost,S) - \varepsilon$$

Large neighborhood search on dTP



- Look for a new solution by solving the remaining sub-problem

Large neighborhood search

- Exploring the neighborhood

```
Procedure moveLNS(P,S,algorithm)
    // define current sub-problem
    Problem P′ = P $\wedge$ (cost $\leq$ getValue(cost,S) – $\varepsilon$)
    propagate(P′)
    while (not stop())
        VariableSet shuffleSet = defineShuffleSet(P,S)
        foreach variable X | X $\notin$ shuffleSet
            P′ = P′ $\wedge$ (X = getValue(X,S))
        if solve(P′,algorithm,S) succeeds
            stop iteration
```

Selecting shuffle sets

- Select a set:
  - large enough to introduce enough flexibility
  - small enough to reduce the overall problem
  - of inter-dependent variables
  - of ill-assigned variables (an improvement can be expected)
- Vary the types of sets that are shuffled
- Vary the size of sets that are shuffled
  - variable neighborhood search

Shuffle sets for dTP

- A set of clients that are
  - Within short distance of some specific client
  - Visited by the same truck
  - Sharing a common type of goods
  - Visited within a common time frame
  - …

A few hints for LNS with CP

- Use incomplete tree search to speedup the sub-problem solution (e.g. LDS)
- Use strong constraint propagation to reduce the neighborhood exploration
- Compute relaxations to prune non-improving neighbors
- Rather switch neighborhood than fully explore one by backtracking

Outline

1. Introduction
2. A didactic optimization problem (dTP)
   - Motivations for cooperation
3. A zoo of CP / LS hybrids
   - Sequential combination
   - *Master / sub-problem decomposition*
   - Improved neighborhood exploration
   - CP Neighborhood search
   - Large neighborhood search
   - Local moves during construction
   - *Local moves over a heuristic*

Local Search and Greedy Construction

- Local search is most often applied to complete solutions
- First build a solution, then improve it
- Idea: better repair while building than afterwards.

=> Incremental Local Optimization

Incremental Local Optimization

- The greedy algorithm makes a mistake at step *n*
- The mistake is discovered at step *n+k*
- Try to repair the steps *n .. n+k*
- Resume the greedy construction at step *n+k+1*

ILO for general CSPs

A simple incomplete method:

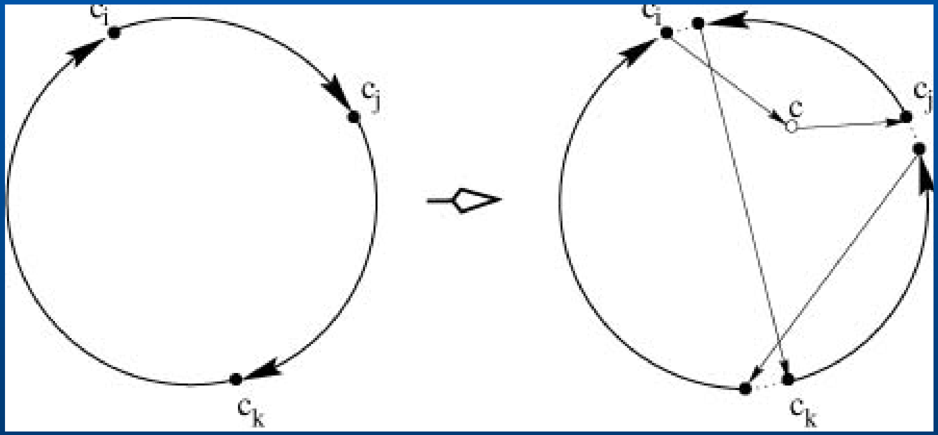
- For a variable ordering $v_1 \ldots v_n$
- Compute a lower bound *lb*
- Start assigning variables
- Choose the value a_{ik} such that $v_i = a_{ik}$ yields the least increase in *lb*
- Whenever *lb* strictly increases,
  - keep $v_i = a_{ik}$,
  - un-assign all variables linked to v_i and
  - try to re-assign them to find the least increase for *lb*

ILO illustrated on dTP

- Enriched greedy construction scheme:
  - Place clients on a stack
  - Insert them one by one minimizing the insertion cost. For client *i*, instantiate
    - $visitedBy_i$
    - $succ_i$
  - Apply local optimization on the truck assigned to *i*
    - Change the order of visits *j* (forall $j \mid visitedBy_j = visitedBy_i$)
  - If an improving sub-route is found, change it

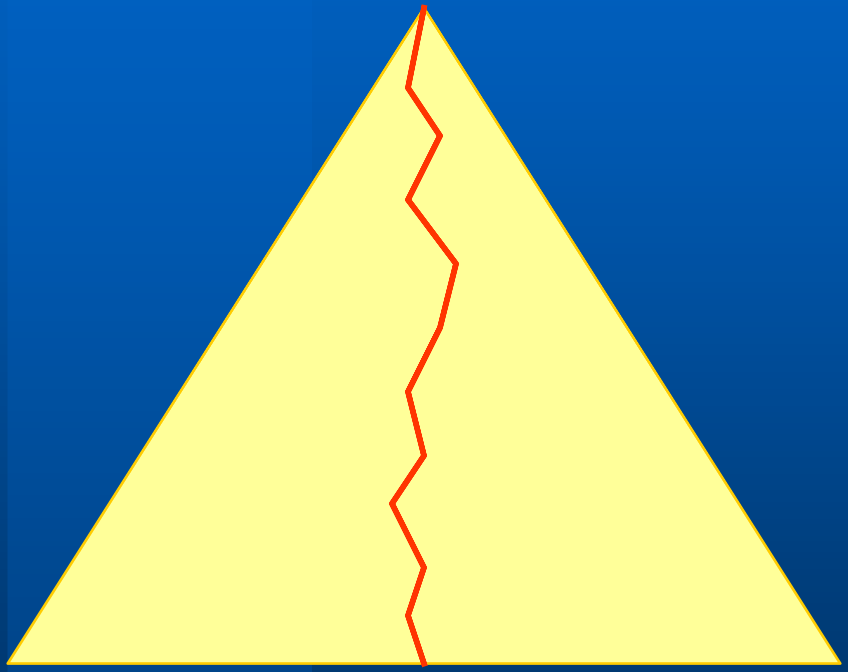
GENeralized Insertion in CP

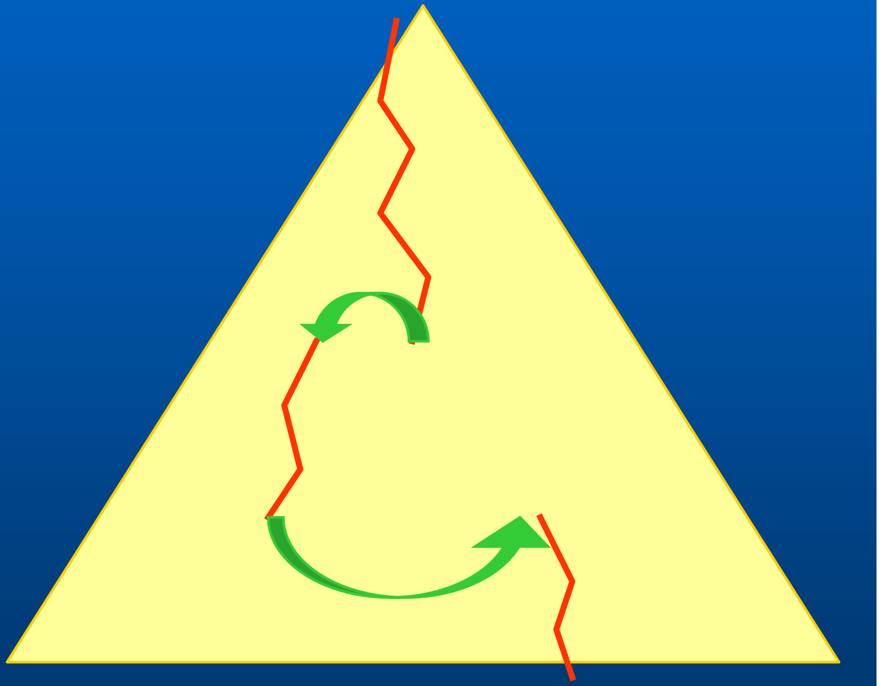
- Allows insertion between non-adjacent customers
- Performs a local optimization simultaneously with the insertion



- c_i c_j and c_k are defined as finite domain variable and their value are identified thru the solution of specific constraint programming model link to the original routing model

Illustration on a search space

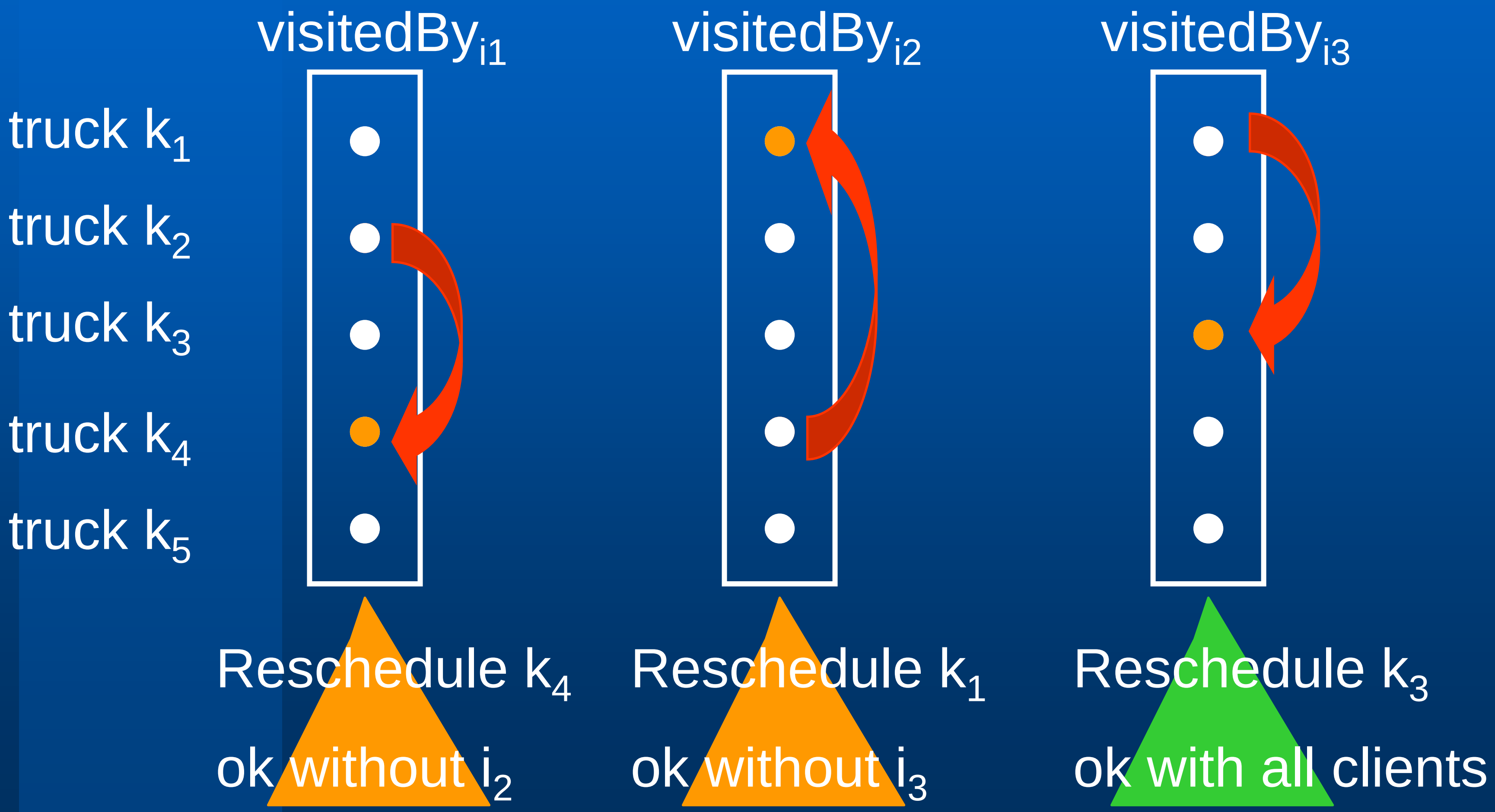




Ejection chains during a greedy process

- Recursive version of the ILO approach
  - For a variable ordering $v_1 \ldots v_n$
  - Choose the value a_{ik} such that $v_i = a_{ik}$ yields the least increase Δlb in lb
  - When $\Delta lb > 0$, un-assign some variable v_l so that lb decreases
  - Reassign v_l to some other value
  - Go-on un-assigning / re-assigning past variables until the least increase in lb is found

Ejection chains on dTP



visitedBy$_{i1}$ visitedBy$_{i2}$ visitedBy$_{i3}$

truck k$_1$

truck k$_2$

truck k$_3$

truck k$_4$

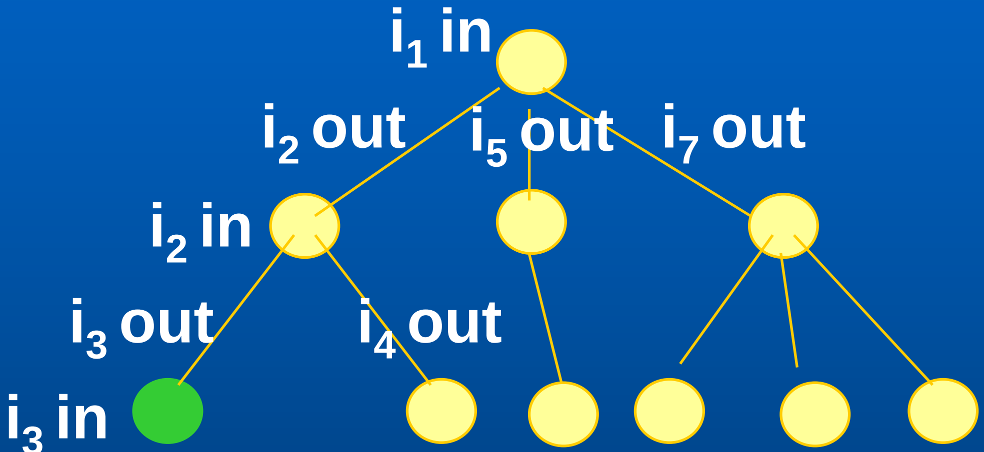
truck k$_5$

Reschedule k$_4$ ok without i$_2$

Reschedule k$_1$ ok without i$_3$

Reschedule k$_3$ ok with all clients

Finding a good ejection chain



i_1 in

i_2 out i_5 out i_7 out

i_2 in

i_3 out i_4 out

i_3 in

- Search for the smallest ejection chain in breadth first search
- Similar to the search for augmenting paths (flows)

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    - *Local moves over a heuristic*

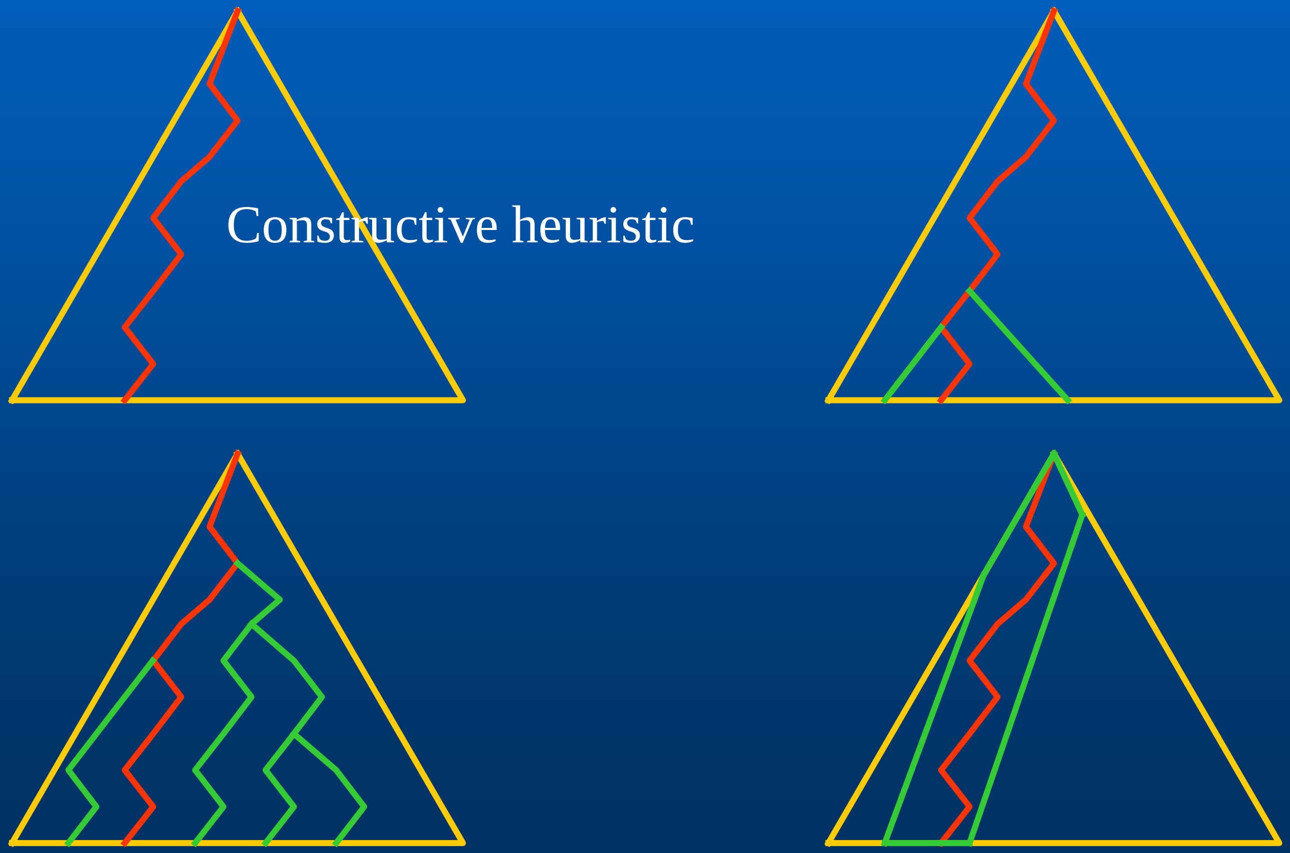
Local moves over a heuristic

- LS is defined as variations over a solution.
- LS can also be applied over an encoding of a solution
  - For a greedy CP method, the search heuristic itself is an encoding
  - Idea: instead of exploring the whole tree, explore variations of the constructive heuristic.

Local search over a heuristic

- Two families of methods
  - Local moves over a value ordering heuristic
    - Restricted candidate lists
    - GRASP
    - Discrepancy based search
  - Local moves over a variable ordering heuristic
    - List scheduling heuristics
    - Preference-based programming

Local search over the value selection heuristic

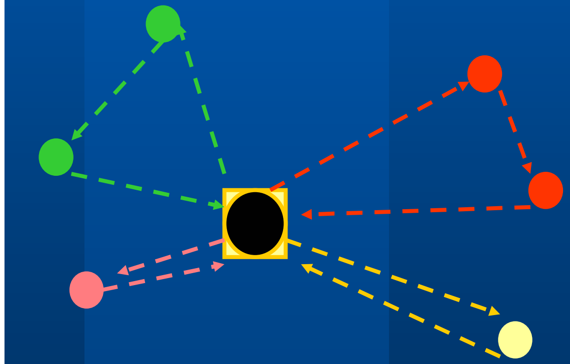


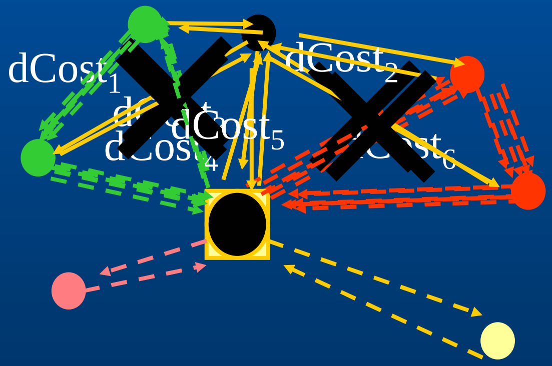
Restricted candidate list

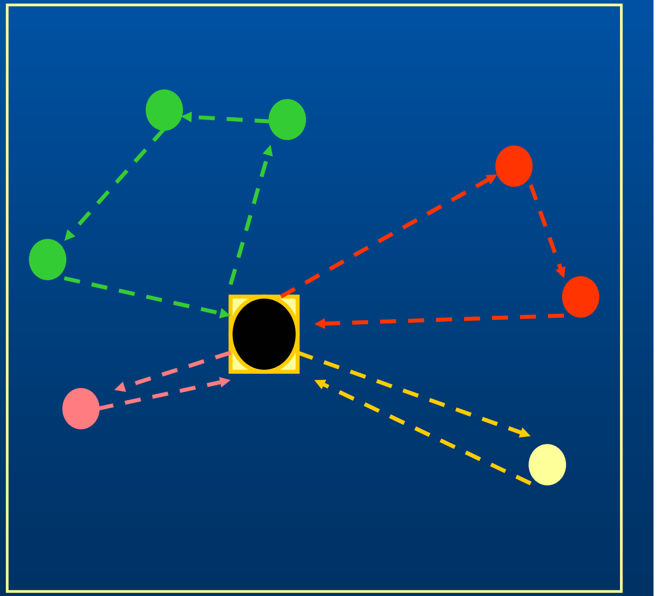
- At each choice point a function *h* is evaluated for all possible choices:
  - the *k* “worst” choices (with high value for *h*) are discarded
  - the choice that minimizes *h* is considered as preferred decision
  - the preferred decision is taken, the remaining choices are taken upon backtracking

Restricted candidate list

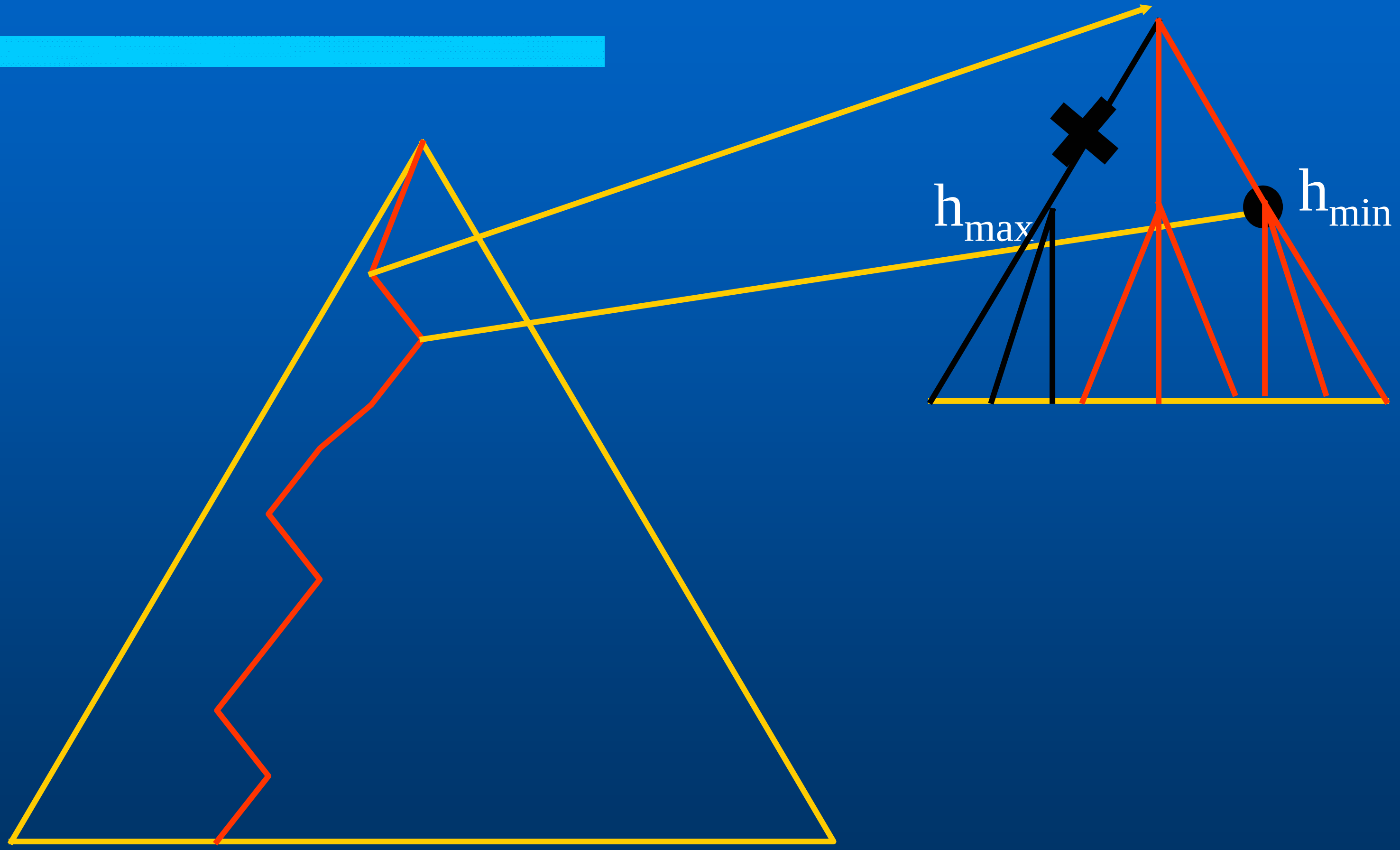
●●●● …







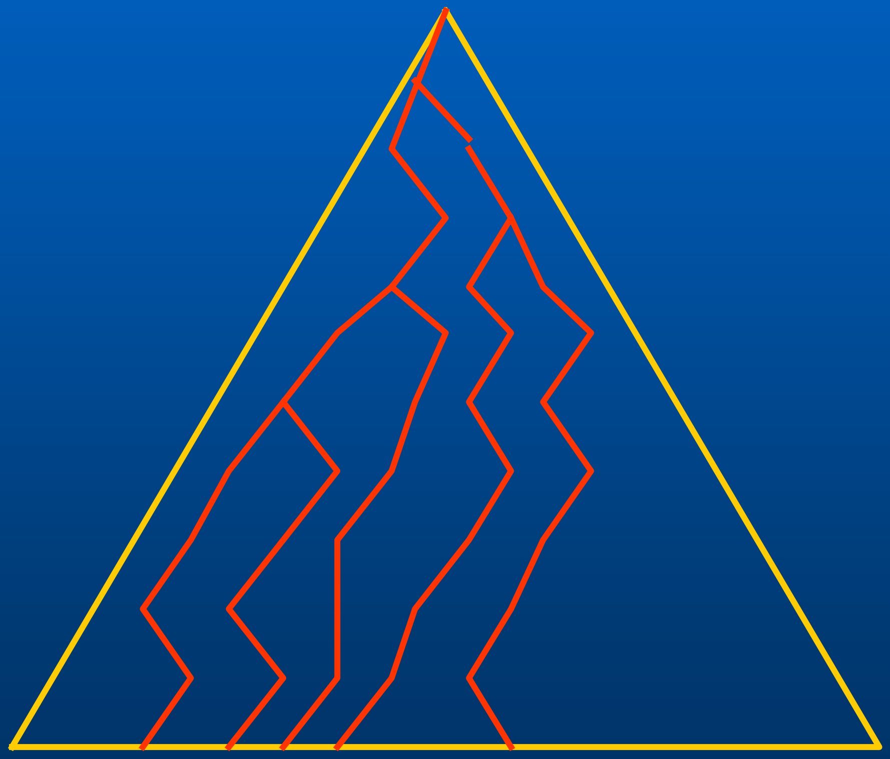
Restricted candidate list



GRASP

- “Greedy Randomized Adaptative Search Procedure”
- At each choice point a function h is evaluated for all possible choices:
  - the preferred decision is chosen by a random function biased towards choices having small value for *h*
  - the preferred decision is taken
  - the process is iterated until a stopping condition is met

GRASP

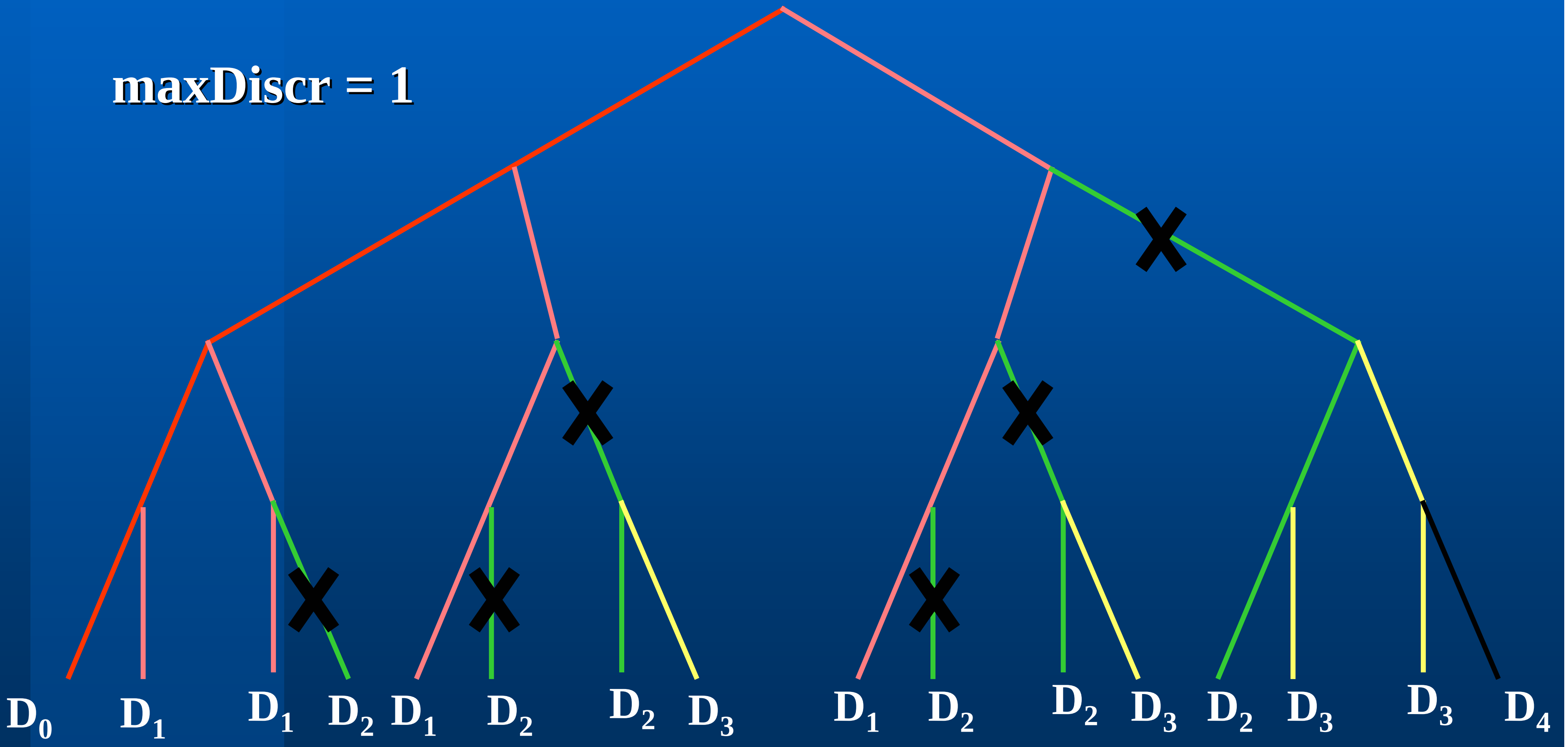


Discrepancy based search

- Idea: good solutions are more likely to be constructed by following always but a few times the heuristic
  - during search, count the number of times the heuristic is not followed (number of *discrepancies*)
  - a maximal number of discrepancies is allowed when generating solutions in the tree.

Discrepancy based search

maxDiscr = 1



CP + some LS: putting things together

- Example:

```
Procedure solve(P)
    while (not stopping condition)
        Solution S = $\emptyset$
        int failLimit = 50
        bool result = solveGRASP(P,S,failLimit)
        if (result)
            P = (P $\wedge$ (Cost(P) < Cost(S))
```

CP + some LS: putting things together

- Example:

```
Procedure solveGRASP(P,S,failLimit)
  while (unscheduled clients exist
          and failLimit not reached)
    Client v = selectVariable(P,S)
    discardBadValues(P,S) //Restricted Candidate List
    InsertionPosition position = evaluateRandom(P,S,v)
    try (insert(P,S,v, position) OR
          notInsert(P,S,v, position))
```

Local search over the variable selection heuristic

- In some problems, a solution can be described by a variable ordering
  - Natural value ordering heuristics
- Examples:
  - List-scheduling heuristics
  - Configuration problems
- Local moves can be applied on the variable sequence itself

Local moves on a heuristic

- Standard process in Genetic Algorithms:
  - Encode the solution
  - Apply local changes to the encoding
  - Construct the new solution (can be done by a CP-based solver)
- In CP: Preference-based programming

Preference-based programming

- Example on Job-Shop scheduling:
  - Consider a ordered list of tasks (priority list)
  - Choice point: (Schedule asap OR Postpone)
    - Take one task at a time from the list and schedule it at its earliest start time
    - otherwise “postpone” the decision on the task for later
  - Local moves on the preferred list of tasks generate different schedules
  - Use tree search to explore a neighborhood of the preferred list

Conclusion

Real life combinatorial optimization problems often require crafting hybrid optimization methods:

- local search is a technique that can complement CP
- many hybrids are possible

« Is it cookery or alchemy ? » M. Wallace

Recipes and tools are emerging …