ON SAT-BASED APPROACHES FOR MULTI-AGENT PATH FINDING WITH THE SUM-OF-COSTS OBJECTIVE

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PRESENTATION STRUCTURE

- Introduction
  - Problem Definition
    - Makespan Optimal Model
      - Sum of Costs Optimal Model
        - Model 1
          - Model 2
            - Reduction of Used Variables
              - Experiments

INTRODUCTION

MAPF – multi-agent path finding

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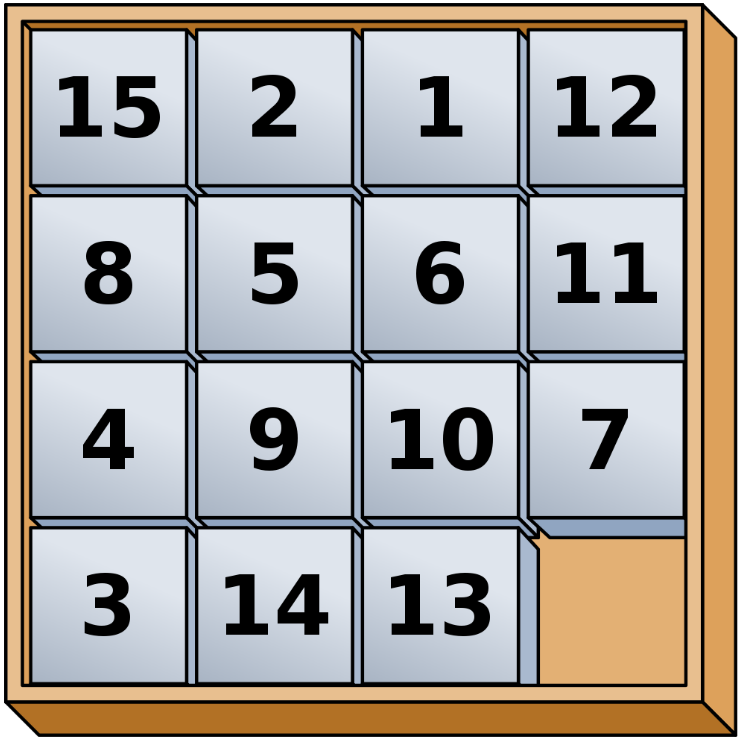

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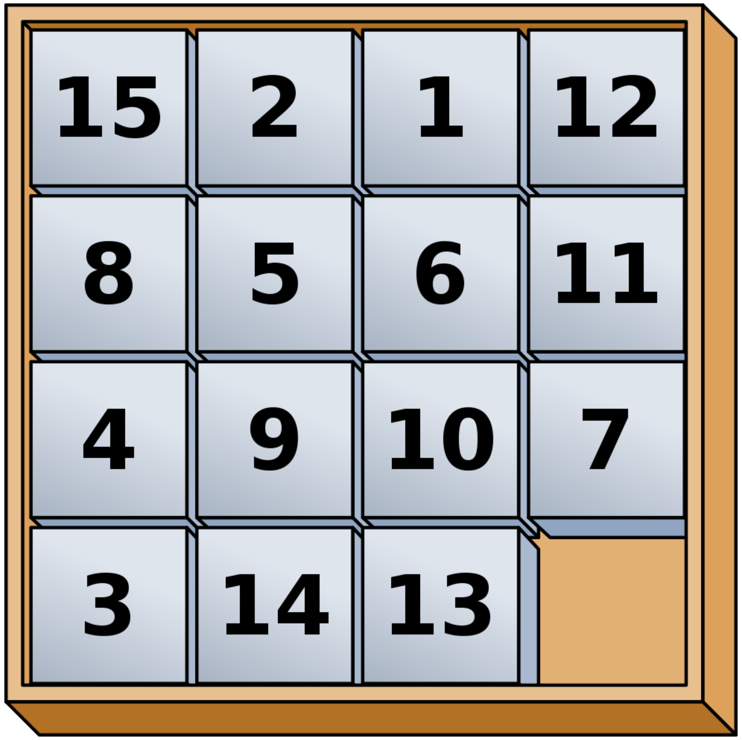

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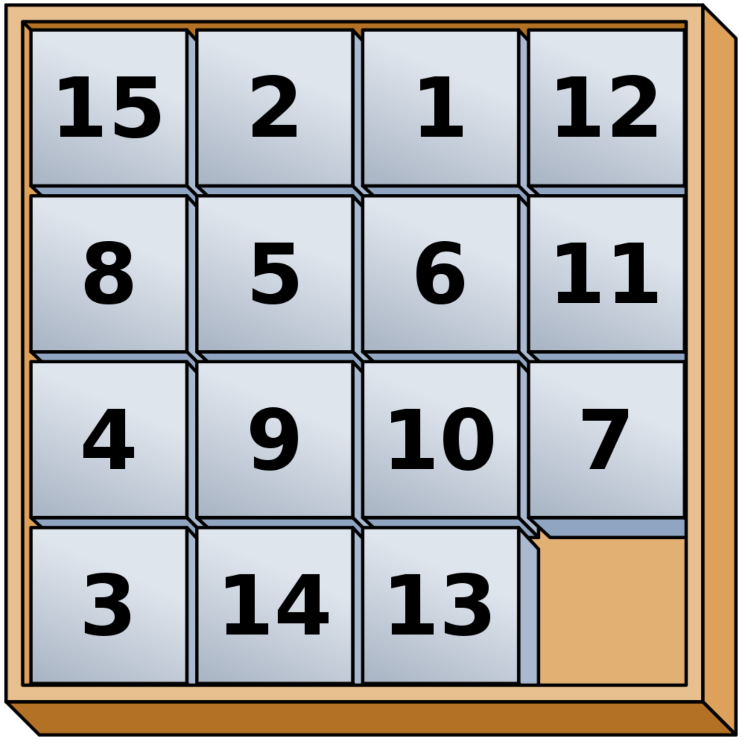

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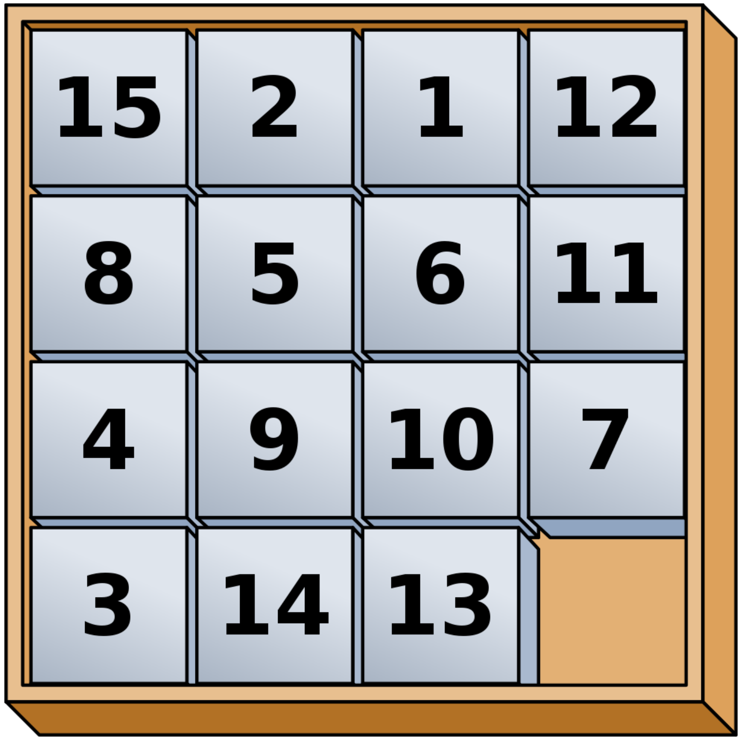

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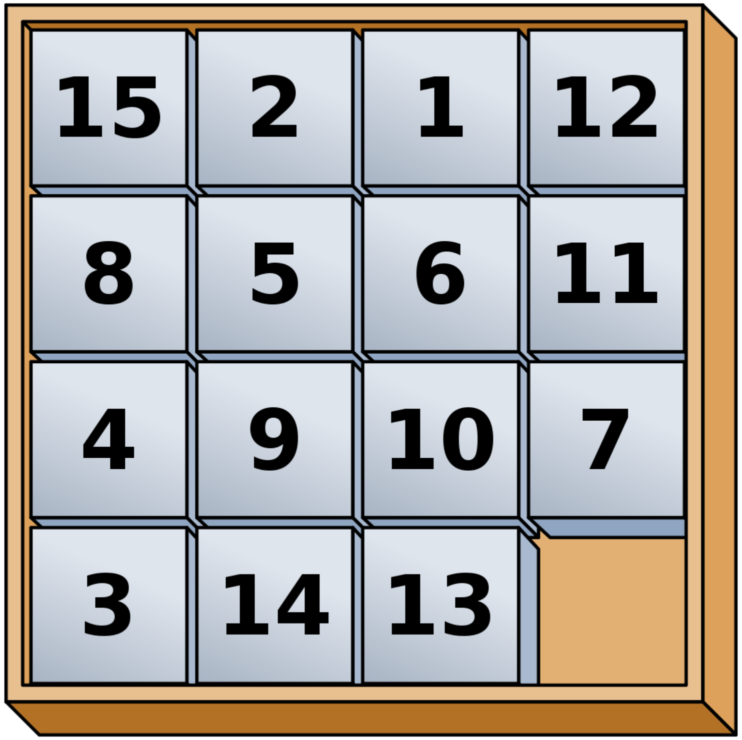
- optimality:
  - Makespan
  - Sum of Costs



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- task – find valid plan for each agent
  - sequence of actions
  - sequence of locations

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Note: train allowed

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Cost functions:

- Makespan $$Mks(\pi) = \max_{i=1\ldots n} |\pi_i|$$
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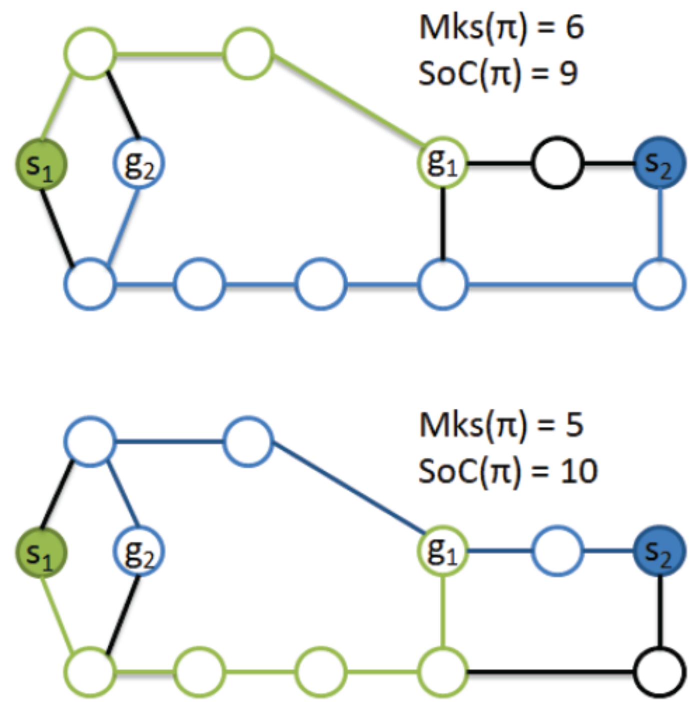

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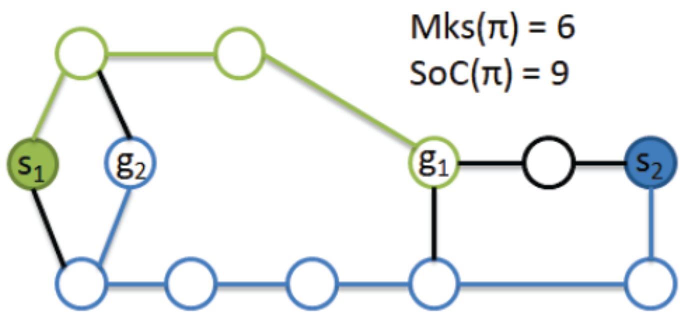
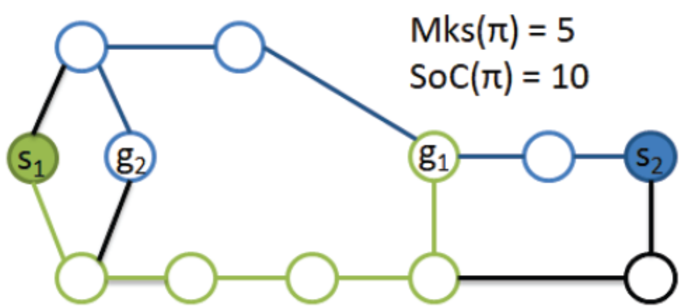
- Feasible solution – polynomial
- Optimal solution – NP-Hard





MAKESPAN OPTIMAL MODEL

- path length is unknown in advance
  - restricted plan length
  - iterative increasing

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- MAPF to SAT

$$\forall x \in V, \forall a \in A, \forall t \in \{0, \dots, T\}: \ At(x, a, t)$$

$$\forall (x, y) \in E, \forall a \in A, \forall t \in \{0, \dots, T-1\}:$$

$$Pass(x, y, a, t)$$

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$Pass(x, y, a, t)$

Note: $\forall x \in V$: $(x, x) \in E$ – *wait*

$$\forall a \in A : At(s_a, a, 0) = 1 \quad (1)$$

$$\forall a \in A : At(g_a, a, T) = 1 \quad (2)$$

$$\forall a \in A, \forall t \in \{0, \dots, T\} : \sum_{x \in V} At(x, a, t) \leq 1 \quad (3)$$

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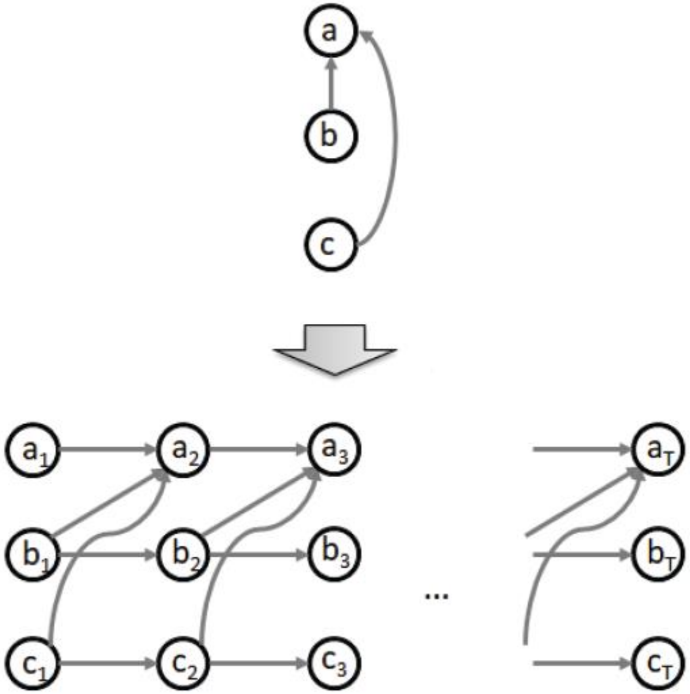
$$\forall x \in V, \forall a \in A, \forall t \in \{0, \dots, T-1\} :$$
$$At(x, a, t) \implies \sum_{(x,y) \in E} Pass(x, y, a, t) = 1 \quad (5)$$

$$\forall (x, y) \in E, \forall a \in A, \forall t \in \{0, \dots, T-1\} :$$
$$Pass(x, y, a, t) \implies At(y, a, t+1) \quad (6)$$

$$\forall (x, y) \in E : x \neq y, \forall t \in \{0, \dots, T-1\} :$$
$$\sum_{a \in A} (Pass(x, y, a, t) + Pass(y, x, a, t)) \leq 1 \quad (7)$$

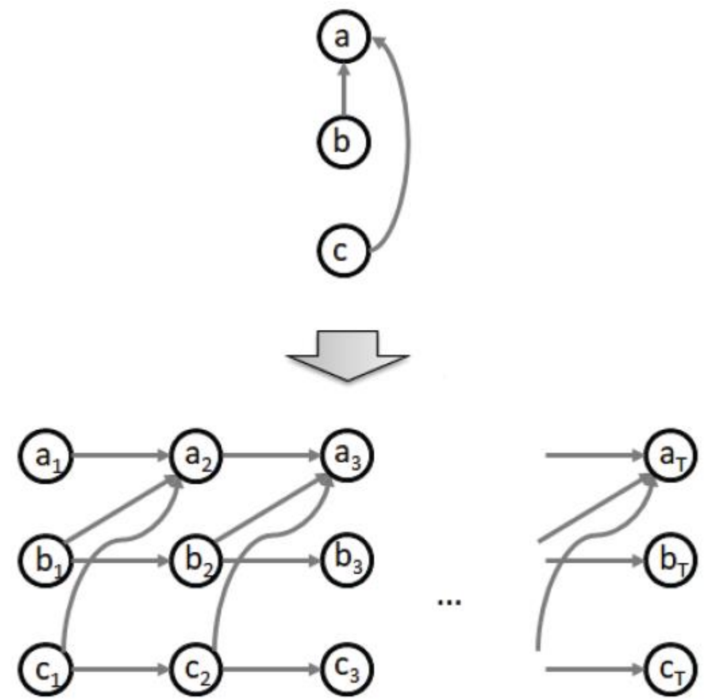
MAKESPAN OPTIMAL MODEL

- SAT representation
  - time-expanded graph



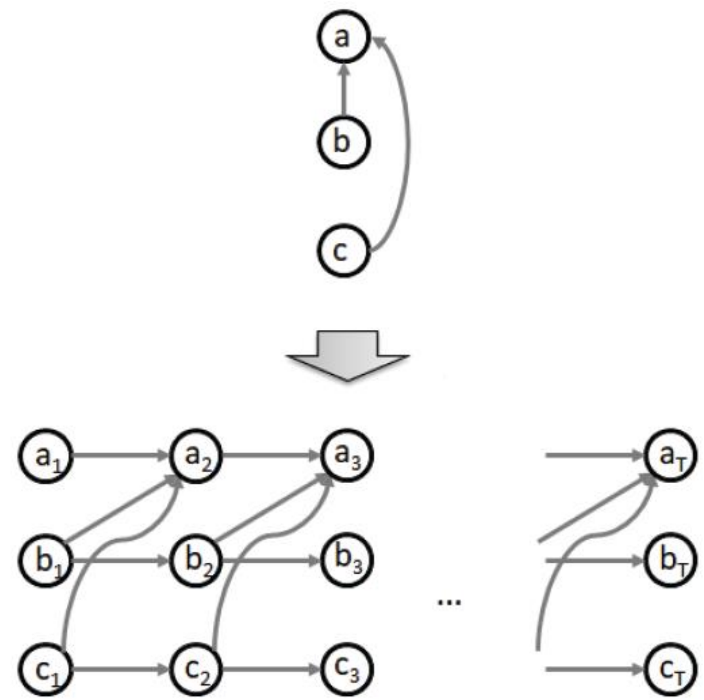
MAKESPAN OPTIMAL MODEL

- SAT representation
  - time-expanded graph
- lower bound
  - $LB(Mks) = \max_{i \in A} SP_i$
  - longest shortest path



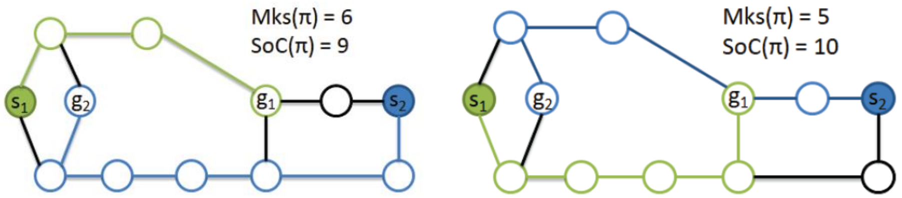
MAKESPAN OPTIMAL MODEL

- SAT representation
  - time-expanded graph
- lower bound
  - $LB(Mks) = \max_{i \in A} SP_i$
  - longest shortest path
- preprocessing for variables
  - some vertices of time expanded graph are for agent unreachable



SUM OF COSTS OPTIMAL MODELS

- Makespan approach won't work



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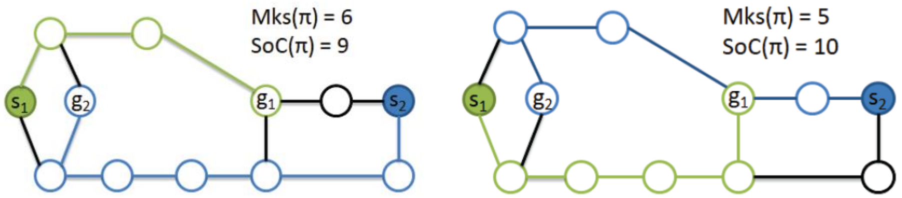

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$$SoC(\pi) \leq UB(SoC) \quad (8)$$

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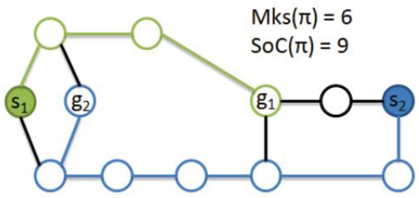

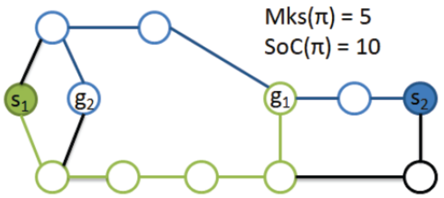

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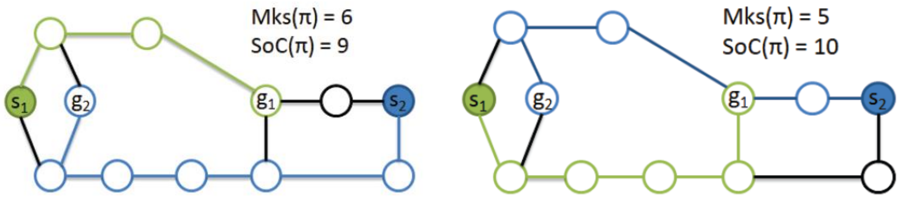

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- to obtain plan with lowest SoC in specified interval

MODEL 1
SUM OF COSTS OPTIMAL MODELS

solve_MAPF(T, C)
generates SAT model with:

- constraints 1-7
- Makespan T
- C as UB(SoC)

Algorithm 1 Model 1

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- simultaneously adds
  - layers of time-expanded graph
  - available actions

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SUM OF COSTS OPTIMAL MODELS

Theorem 1. *If there exists a solution with the Sum of Costs* $LB(SoC) + \delta$ *then this solution can be found in a time-expanded graph with* $LB(Mks) + \delta$ *layers.*

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□

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SUM OF COSTS OPTIMAL MODELS

problems with algorithm 1:

- iterates too many makespans
- at the end final makespan is lager than needed

Algorithm 1 Model 1

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- find makespan that guarantees existence of optimal solution

  $LB(SoC) = \sum_{i \in A} SP_i$
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  while No Solution **do**
    $SoC \leftarrow$ opt_MAPF($LB(Mks) + \gamma,$
      $LB(SoC), |A| * LB(Mks) + \gamma$)
    $\gamma \leftarrow \gamma + 1$
  end while
  $\delta \leftarrow SoC - LB(SoC)$
  opt_MAPF($LB(Mks) + \delta, LB(SoC), SoC$)
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Algorithm 2 Model 2

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SUM OF COSTS OPTIMAL MODELS

1. optimal Makespan is found
   - with no restriction on Sum of Costs

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1. optimal Makespan is found
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2. computes δ by Theorem 1

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1. optimal Makespan is found
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2. computes δ by Theorem 1
3. finds optimal solution

Algorithm 2 Model 2

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function MODEL 2
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    LB(Mks) = max_{i∈A} SP_i
    LB(SoC) = Σ_{i∈A} SP_i
    γ ← 0
    while No Solution do
        SoC ← opt_MAPF(LB(Mks) + γ,
                LB(SoC), |A| ∗ LB(Mks) + γ)
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    end while
    δ ← SoC − LB(SoC)
    opt_MAPF(LB(Mks) + δ, LB(SoC), SoC)
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- Makespan T
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```
function MODEL 2
    ∀a_i ∈ A : SP_i = shortest_path(s_i, g_i)
    LB(Mks) = max_{i∈A} SP_i
    LB(SoC) = Σ_{i∈A} SP_i
    γ ← 0
    while No Solution do
        SoC ← opt_MAPF(LB(Mks) + γ,
                 LB(SoC), |A| ∗ LB(Mks) + γ)
        γ ← γ + 1
    end while
    δ ← SoC − LB(SoC)
    opt_MAPF(LB(Mks) + δ, LB(SoC), SoC)
end function
```

- optimal solution found using (9) in interval:

$$\langle LB(SoC), |A| * LB(Mks) + \gamma \rangle$$

$$Minimize\_SoC(LB(SoC), UB(SoC)) \quad (9)$$

MODEL II

SUM OF COSTS OPTI

- with best Sum of Cost from all optimal Makespans
- any feasible Makespan is sufficient at this stage, but nonoptimal is costly

Algor

1. optimal Makespan is found
   - with no restriction on Sum of Costs
2. computes δ by Theorem 1
3. finds optimal solution

opt_MAPF(T, L, U)
generates SAT model with:

- constraints 1-7
- Makespan T
- L as LB(SoC)
- U as UB(SoC)

w

$SoC \leftarrow$ opt_MAPF$(LB(Mks) + \gamma,$
$LB(SoC), |A| * LB(Mks) + \gamma)$
$\gamma \leftarrow \gamma + 1$
end while
$\delta \leftarrow SoC - LB(SoC)$
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SUM OF COSTS OPTIMAL MODELS

time-expanded graph represented by SAT model can be seen as if each agent had his own version of this graph and those were connected by constraints

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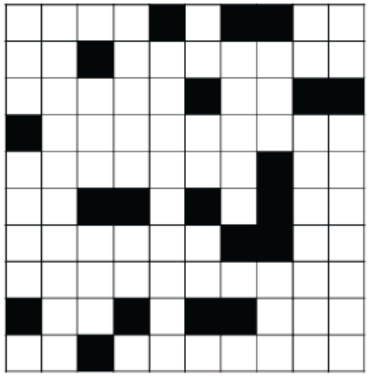
- for each agent is created separate time-expanded graph with $SP_i + \delta$ layers
- such graphs are interconnected by constraints 1-7
- in this way, agents with short paths would disappear from abstract graphs of other players once they reach goal
  - forbid their goal after timestep $SP_i + \delta$

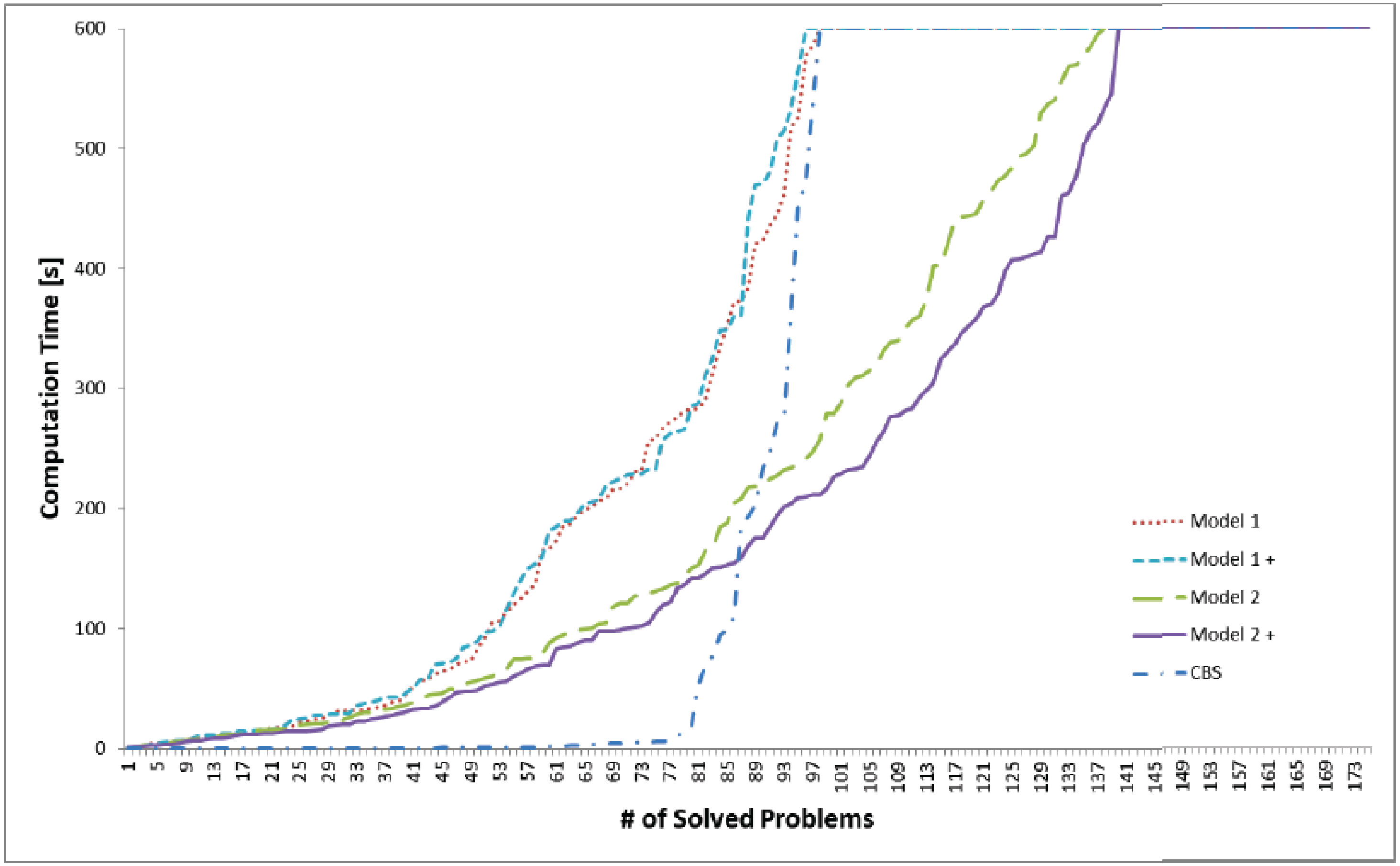
EXPERIMENTS

- 2d grids 8x8 up to 16x16
- 20% of the cells are impassable
- 1-2x grid width agents
- randomly generated unique start and unique end positions



- each setting 5x
- altogether 175 unique instances

EXPERIMENTS



EXPERIMENTS

	M. 1	M. 2	M. 1+	M. 2+	CBS
# of solved	97	137	95	**139**	97
# of fastest	0	4	3	46	**88**
# of fastest (without CBS)	9	8	6	**118**	–
IPC score	16.11	44.56	16.76	57.07	**92.50**
IPC score (without CBS)	57.19	110.54	53.93	**134.29**	–

KEYWORDS

- MAPF
  - SAT representation
    - Time-expanded graph
      - Sum of Costs
        - Makespan
          - Model 1
            - Theorem 1
              - Model 2
                - Reduction of Used Variables